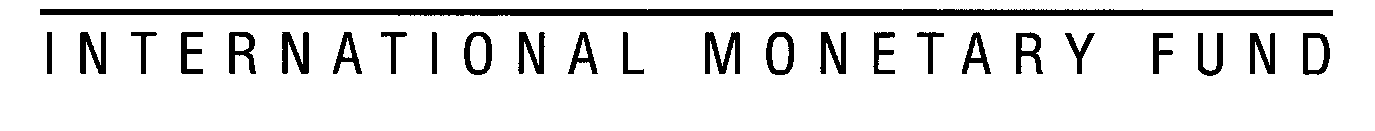
Systemic Risk From Global Financial Derivatives: A Network Analysis of Contagion and Its Mitigation With Super-Spreader Tax

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Abstract

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| Financial network analysis is used to provide firm level bottom up holistic visualizations of interconnections of financial obligations in global OTC derivatives markets. This helps to identify systemically important financial intermediaries (SIFIs), analyse the nature of contagion propagation and also to monitor and design ways of increasing robustness of the network. Based on 2009 FDIC and individually collected firm level data covering gross notional, gross positive (negative) fair value and the netted derivatives assets and liabilities for 202 financial firms which includes 20 SIFIs, the bilateral flows are empirically calibrated to reflect data based constraints. This produces a tiered network with a distinct highly clustered central core of 16 SIFIs that account for 78% of all bilateral exposures and a large number of FIs on the periphery. The topology of the network results in the “too interconnected to fail” (TITF) phenomenon in that the failure of any member of the central tier will bring down other members with the contagion coming to an abrupt end when the ‘super-spreaders’ have demised. As these SIFIs account for the bulk of capital in the system – *ipso facto* no bank among the top tier can be allowed to fail highlighting the untenable implicit socialized guarantees needed for these markets to operate at their current levels. Systemic risk costs ofhighly connected SIFIs nodes are not priced into their holding of capital or collateral. An eigenvector centrality based ‘super-spreader’ tax has been designed and tested for its capacity to reduce the potential socialized losses. |

JEL Classification Numbers:JEL: G01, G21, G17, G32, G15

Keywords: Systemic Risk; Financial Network; Too Interconnected to Fail; Eigenvector Centrality; Super Spreader Tax

Table of Contents  
1.Introduction   
 1.1Background  
 1.2 SIFIs in Derivatives Markets and Market Concentration   
 1.3 Market Data Based Systemic Risk Measures and Financial Network Perspective  
 1.4 Structure of the Paper

2. Financial Network Analysis   
 2.1 a Adjacency Matrix and Gross Flow Matrix For Derivatives   
 2.1.b Bilaterally netted Matrix of Payables and Receivables  
 2.2 Topology of Financial Networks: Complete, Random, Core-Periphery, Clustered and Small  
 World   
 2.3 Economics Literature on Financial Networks  
 2.4 Eigenvalue Perspective of Network Stability

3. Contagion and Stability Analysis   
 3.1 Furfine (2003) Methodology : Cascades from Failure of a Trigger Bank   
 3.2 Financial Network Stability Analysis: The Eigen-Pair  
 3.3 Mitigation and Management of Financial Contagion: Design of Super-spreader Tax

4. Empirical Results: Network Analysis of the Calibrated Aggregated Global Derivatives  
 Market   
 4.1Empirical (Small World) Core-Periphery Network Algorithm   
 4.2 Global Derivatives Network Statistics (2009 Q4)   
 4.3 Network and Furfine Stress Test Results: *Too Interconnected to Fail Quantified*  
 4.4 Right Eigenvector Centrality and Quantification of Systemic Risk Index of FIs   
 4.5 Evaluation of the Super-spreader tax (2009 Q4)  
  
5. Conclusion   
6. References  
Appendix

# Introduction

**1.1Background**

Systemic risk from financial derivatives markets came to the forefront with the accelerated growth in credit derivatives to about $60 trillion at its peak in 2007 and with it the excessive liabilities on a small segment of credit default swaps (CDS) on residential mortgage backed securities of key institutions such as American Insurance Group (AIG). The collapse of the US house prices triggered widespread weakness in this class of credit derivatives which had initially helped in the proliferation of US mortgage backed securities globally and subsequently led to the unprecedented tax payer bailouts of financial institutions estimated at over $14 trillion over the course of the 2007/8 financial crisis, Alessandri and Haldane (2009). The bailouts premised a moral hazard problem identified as arising from some financial institutions being ‘*too interconnected to fail’ (TITF)*. This refers to the larger threats to the financial system as a whole that follow from domino effects of the failed entity on others. The AIG bailout aimed to stem the negative externalities of its failure to deliver on its CDS guarantees to major counterparties.[[2]](#footnote-3) The correlated jump to default spikes in CDS premia for contracts for which Lehman Brothers was the protection seller and the triggering of CDS on Lehman Brothers as the reference entity is estimated to have magnified liquidity demands[[3]](#footnote-4) during the conditions which led to a ‘run’ on the wholesale money markets in 2008, Gorton and Metrich (2009).

The objective of this paper is to quantitatively model the contagion like threats posed by the activities of large financial intermediaries in the over-the-counter (OTC) derivatives markets. The size of gross notional amounts outstanding of OTC derivatives globally is estimated by BIS Statistics at over $614 trillion in December 2009.[[4]](#footnote-5) Of this, in Q4 2009 the gross notional of derivatives traded by FDIC US financial institutions is $212.8 trillion and closer to $ 285 trillion when commercial banks and others are included as holding companies.[[5]](#footnote-6) The exchange traded derivatives, in contrast, are valued at no more than about $73.14 trillion[[6]](#footnote-7) for Q4 2009 and these are mostly futures and options contracts. When compared to the size of world GDP at $70 trillion, and size of the global bond market (total debt outstanding) at about $82 trillion, the implication is that the size of off balance sheet activities of financial intermediaries (FIs) has grown to many multiples of their assets. Derivatives obligations of FIs under conditions of market wide adverse movements on the underlying (such as interest rates, house prices, exchange rates, corporate debt and equity and sovereign debt) could overwhelm the equity and assets of FIs.

**Figure 1: Gross Notional of Financial Derivatives** ($billions, BIS, June 2010 Quarterly Review)

**Figure 2: Gross market values OTC Derivatives ($ billions, BIS, June 2010)**

Figures 1 and 2 give the gross notional and the gross market value of the global derivatives in the 5 different derivatives products. Post 2008, there has been a considerable decline in both gross notional and gross market value due to compression and tear ups of bilateral positions. Gross notional is the best measure of the size of derivatives activity and the gross market value gives an estimate of the economic risk in derivatives arising from the volatility of underlying reference/asset prices, leverage and hedge ratios, duration, liquidity, and counterparty risk. Toward, the latter part of 2010, the increase in gross market value of derivatives can be attributed to growing volatility in financial markets.

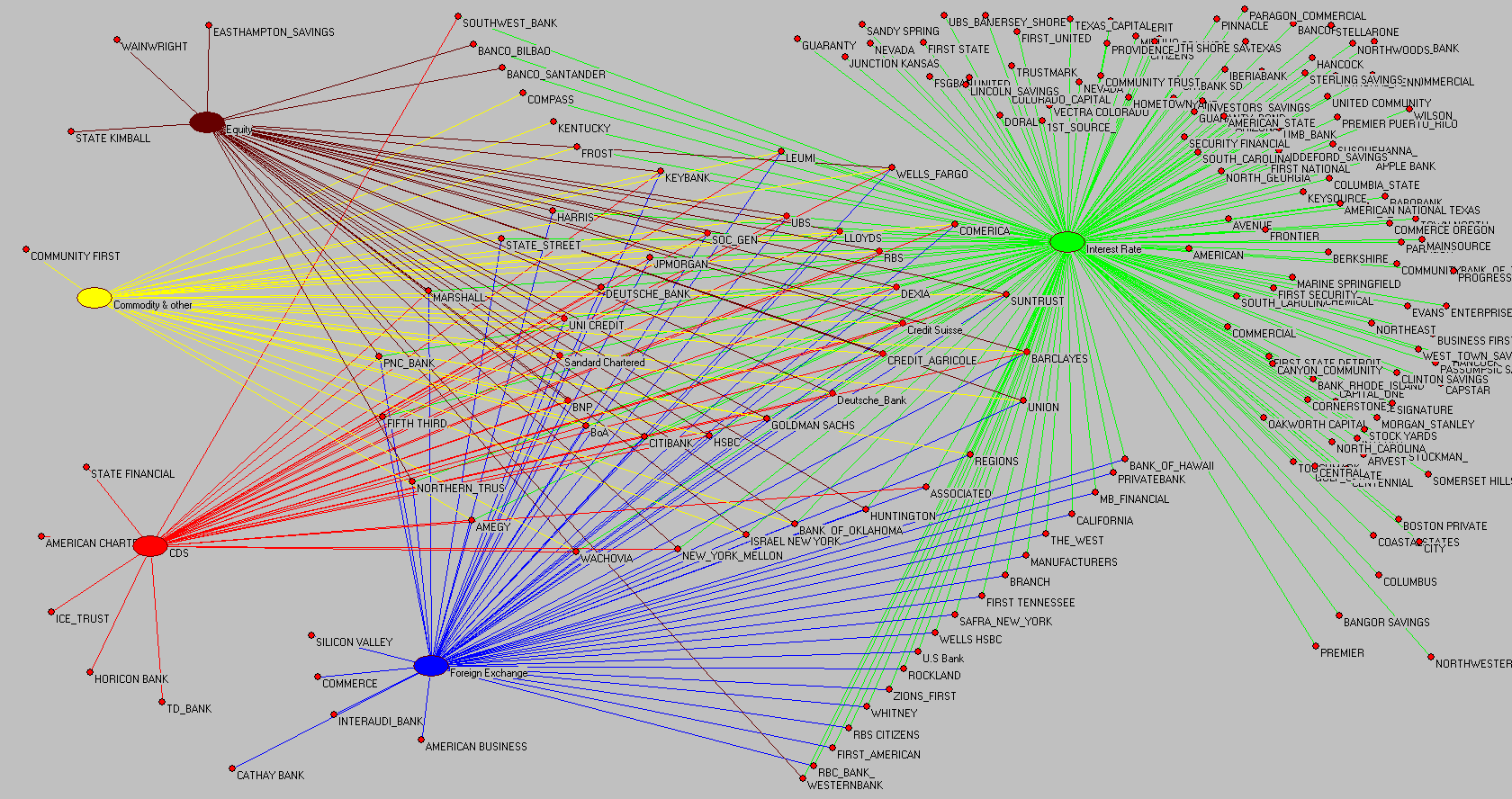
**1.2 SIFIs in Derivatives Markets and Market Concentration**

Typically risk sharing in advanced financial systems is accomplished by OTC derivatives. At the level of the individual user these schemes appear plausible as a means of risk shifting, but at the macro-level they may lead to systemically unsustainable outcomes as interconnectivity of FIs increases. The success of risk sharing at a system level is crucially related to the numbers of protection sellers or those who have a considerable position in derivatives liabilities and the structural interconnections involved in the provision of guarantees. Indeed, the key structural aspect of the networks underpinning financial derivatives has been summarized in the 2009 Fitch survey[[7]](#footnote-8): “dependence on a limited number of counterparties looks to be a permanent feature of the market; this is underscored by the fact that the top 12 counterparties[[8]](#footnote-9) comprised 78% of total exposure in terms of the number of times cited, up from the 67% reported last year. The top five institutions that provided volume figures accounted for 95% of total notional amount bought and sold. This concentration is a reflection of the dominant role of banks and dealers as counterparties, particularly after the collapse of a limited number of financial institutions who were important intermediaries in this market.”

In an attempt to address systemic risk in the financial system, the notion of a systemically important financial intermediary (SIFI) has come to the forefront. Joint statements[[9]](#footnote-10) by the Financial Stability Board, IMF and BIS have identified these FIs in regard to large size, prominence in markets or functions (non-substitutability), complexity, global activity and interconnectedness. It is considered important that some metric for systemic risk based upon the impact of a firm’s failure on the financial system as a whole is found and already some figures for extra capital charges have been suggested. They have been stipulated to range from 1% to 2.5% depending on the assessment of the above factors for SIFIs and an additional 1% capital charge may be levied on the institutions considered to be very risky. The question is: can SIFIs internalize their systemic risk costs arising from their activities in specific sectors and markets with a non-specific capital surcharge? In what way can the surcharge be attributable to a SIFI’s strategies, for instance, in global derivatives markets which have been a cause for concern?

This study involves 202 participants of derivatives markets based in the US and Europe. Of these 20 are global SIFIs and 18 of them are broker dealers in over 4 of the 5 derivatives markets. Using an affiliation graph in Figure 3 of the number of participants in each of the derivatives products, the dominance of 8 broker dealers can be seen in the centre of Figure 3. The majority of 126 belong to only 1 product market; 23 to two products only; 8 to three, 10 to 4 and 8 to all five markets. Affiliations in only 1 market are arranged at the outer corners of the graph in Figure 3, those in any 2 markets are arranged further in from the edges and the most central circle consists of the 8 broker dealers in all 5 markets surrounded by those who are present in 4 markets.

**Figure 3: Affiliation Graph of Global SIFIs and US-FDIC FIs as Participants in the Five Financial Derivatives Markets** (Total number of participants is 202 based on gross notional data for FX (Blue: 48), Interest Rate Swaps (Green: 189), Equities (Brown: 25), Commodities and others (Yellow:22); For CDS (Red: 23) (Source: FDIC Call Reports Q4 2009 for US FIs, Data on global SIFIs from individual Annual Financial Statements)



It is important to study the systemic risk from activities of SIFIs and other FIs in the different derivatives markets to fully understand the propagation of financial contagion which more often than not emanates from specific markets and underlying assets (eg. mortgage backed securities in the CDS market, see Markose *et. al.* (2010)). However, the starting point of this study is the calibrated network of bilateral assets and liabilities that the 202 FIs face in respect to derivatives contracts *aggregated* over all derivatives products. This data is summarized in Table 1. As discussed in the OCC Quarterly Report (2009), the first step in measuring credit exposure in derivatives contracts is to identity the contracts which would lead the FI to lose value if the counterparty to the contract defaults. The contracts with gross positive fair value (GPFV) for a FI constitute the derivatives receivables from its counterparties without taking into account netting. Where no legally binding bilateral netting exists between a FI and its counterparties and the FI receives no collateral from them, GPFV is the maximum credit exposure or losses a FI will face if its counterparties fail. In turn, a FI’s derivatives contracts with a negative fair value, with no netting and no collateral posted by the FI to its counterparties, represents gross negative fair value (GNFV) or the total derivatives payables to the FI’s counterparties. The fair values are obtained from the market price of the instrument if it is traded or model determined if it is not traded. Derivatives liabilities and derivatives assets are respectively estimated once the gross payables and gross receivables for each FI are adjusted for collateral and bilaterally netted on a multi-product basis where master agreements exist and summed over all counterparties. The derivatives assets are the amounts that matter as the loss given default of a FI’s counterparties and attract a Basel II capital charge. Regulators such as the US Office of Comptroller of the Currency (see OCC Quarterly Derivatives Trading Reports) have resorted to an aggregate metric called net current credit exposure (NCCE) to evaluate credit risk in banks’ derivatives activities which is based on derivatives assets. However, as first noted by Singh (2010, italics in original) *a FI’s derivative payables is the risk imposed on the rest of counterparties when it fails* and hence this represents the causal factor for a FI’s systemic risk from its activities in the derivatives markets. Hence, in what follows, the focus will be on the network of derivatives liabilities and a systemic risk surcharge on FIs will be determined on the basis of their interconnectivity and centrality in the system from the direction of liabilities of FIs.

**Table1:Value and Market Share of Financial Derivatives for 202 FIs ($ Trillions)**Source: FDIC Call Reports Q4 2009 for US FIs, Data on global SIFIs from individual Annual Financial Statements

|  |  |  |  |
| --- | --- | --- | --- |
|  | **All FIs** | **Top 16 FIs** | **Top 26 FIs** |
| **Gross Notional** | 674.369 | 659.263  (97.76%) | 673. 827  (99.92%) |
| **Gross Positive Fair Value (GPFV)** | 10.465 | 10.254  (97.98%) | 10.453  (99.89%) |
| **Gross Negative Fair Value (GNFV)** | 10.144 | 9.934  (97.92%) | 10.134  (99.89%) |
| **Derivatives Assets** | 1.168 | 1.068  (91.39%) | 1.15  (98.52%) |
| **Derivatives Liabilities** | 0.889 | 0.816  (91.78%) | 0.874  (98.33%) |
| **Total Assets** | 37.252 | 27.863  (74.80%) | 32.837  (88.15%) |
| **Tier 1 capital** | 1.413 | 1.014  (71.60%) | 1.207  (85.48%) |

NB:The market shares are in parenthesis.

Table 1 highlights two aspects of systemic risk characterizing derivatives market: the unique and dominant role played by SIFIs and the large size of derivatives assets and liabilities relative to Tier 1 capital of the sector. Further, there is a growing view that as the underlying of financial derivatives are pro-cyclical to macro-economic factors and guarantors of credit risk are caught up in a self-reflexive loop[[10]](#footnote-11), this can promote an excessive growth of these derivatives which may appear rational at the level of the individual unit, but far from mitigating volatility, this can bring about extreme tail events that the system will be unable to provide protection in regard to settlement liquidity (see, Brock *et. al.* (2009), Rajan (2006)). Not present in Table 1 is the data that shows smaller and smaller end user hedge benefits are supported by larger and larger offsetting activities done by broker dealers of derivatives with a large proportion of payoffs at settlement going to speculative ‘naked’ buyers of derivatives.[[11]](#footnote-12) The dislocation here is that financial derivatives trading begin to provide greater profitability to broker-dealers in the short run than returns from real side investment.  
  
**1.3 Market Data Based Systemic Risk Measures and Financial Network Perspective**  
At least since the NBER paper by Michael Darby (1994), systemic risk from financial derivatives has been considered as a threat due to the sheer size of the OTC markets and the concentration of the market. To this may be added the institutional weakness of OTC derivatives markets, Duffie *et. al.* (2010) and Duffie (2010), and correlated movements (Brunnermeir *et.al,* (2009)) in the underlying assets of financial derivatives, interest rates, exchanges rates, bank assets, corporate and sovereign debt that can exacerbate the size of derivatives liabilities and also trigger multiple obligations across many sectors. In view of the growing *structural* concentration in the provision of risk guarantees through financial derivatives, the fragility of the modern risk sharing institutions are germane to issues on systemic risk. Existing statistical models for systemic risk may fail to identify the threats to stability from the topological structures of financial networks often referred to as *TITF* arising from the concentration of financial links between a few key players. The objective of this paper is to rigorously investigate the empirical topological structures of the financial derivatives markets using network analysis. This can help to identify systemically important players in terms of network connectivity, analyse the nature of contagion propagation and also to monitor and design ways of increasing robustness of the network.

By its nature of being a negative externality, systemic risk implications of a FI’s connectivity and concentration of obligations have to date not been factored into the capital or collateral being held by FIs. Many have remonstrated (see, Acharya and Richardson (2009), Bernanke (2009)) against the socialization of losses from SIFIs and have pointed out the need for highly connected FIs to internalize their systemic risk costs. In a ratings based system, as succinctly pointed out by Haldane (2009), leniency of capital and collateral requirements for a few large highly rated financial intermediaries has resulted in excessive expansion of credit and derivatives activities by them which is far beyond what can be sustained in terms of system stability. Further, the expectation that FIs that are *TITF* will be bailed out also increases their ratings than if this were not to be the case (see, Moodys (2011) and Haldane (2010)). Haldane (2009) calls such highly interconnected FIs ‘super-spreaders’ and we will retain this epithet in the financial network modeling that follows. Haldane (2009) recommends that super-spreaders should have larger buffers. To date, the financial networks literature have produced interesting qualitative analysis for contagion in networks, but have not operationalized a metric for systemic risk for the network system nor a measure of a FI’s contribution to systemic risk.

This paper is novel in adopting the maximum eigenvalue of the weighted bilateral matrix of financial liabilities (given as a ratio of Tier 1 capital of the FI) as the metric for systemic risk. A weighted eigenvector centrality based capital surcharge is obtained from a stability condition determined by the maximum eigenvalue for the untaxed financial network. Significantly, the eigenvector centrality of a FI can be large if its total liabilities and/or or its capital is low and also if it is connected to counterparties with high eigenvector centrality. The super-spreader tax is tested for its efficacy to reduce potential socialized losses from an idiosyncratic failure of a major FI. The capital surcharge can be viewed as a Pigovian externalities tax and it entails a linear parameter that can be scaled upwards so that the desired value for the maximum eigenvalue that increases stability of the financial network is achieved. Thus, the capital surcharge is consistent with the desired reduction in network instability and to each FI’s contribution to it given the topology of the financial network. Note, the power iteration algorithm that solves for the maximum eigenvalue of the weighted network simultaneously solves for the eigenvector centrality of all participants in the financial network. Successful systemic risk stabilization requires that funds from the capital surcharges are held in an escrow account to engineer bailouts of counterparties of a failed FI. Clearly, costs imposed on others by the failure of a FI are less likely to be held voluntarily by the FI than the capital needed to save itself from its own actions. As will be seen, the proposal that FIs are to be taxed in proportion to their right eigenvector centrality in the financial network for derivatives liabilities vitiates the need for any market price data and further only contagion risk from counterparty failure due to the interconnectedness of FI’s derivatives liabilities is considered rather than other indirect costs of failure.

In contrast to the approach taken in this paper, by and large, systemic risk models have relied on statistical market price based data to extrapolate from the micro-prudential risk measures of Basel II to obtain macro-prudential measures. To analyze so called interconnectedness risk between FIs, matrix of bilateral correlations or non-linear copula based co-movements purported to represent extreme market conditions is constructed. The main market-based systemic risk measures that have been proposed are Conditional VaR (CoVaR) by Adrian and Brunnermeier (2009), System Expected Shortfall (SES) Acharya et al. (2010), Co-risk by Chau-Lan (2010), DIP (Distress Insurance Premium) by Huang et al. (2010), POD (Probability that at least one bank becomes distressed) by Segoviano and Goodhart (2009), Shapley-Value by Tarashev *et. al.* (2010) and Macro-prudential capital by Gauthier et. al (2009).

The ingredients of most statistical systemic risk measures involve the following. Firstly, a system wide expectation is formed of joint extreme/tail losses on a debt/liability or equity portfolio of all FIs (joint portfolio) often using VaR or Expected Shortfall type measures. The systemic risk of a FI is then estimated in a number of ways: a simple beta statistic (ratio of covariance of tail loss of FI to the variance of the joint portfolio tail loss); the marginal contribution given as expected loss of the FI conditional on the joint portfolio loss (Huang *et. al* (2010)); as ‘incremental’ (Δ-CoVaR) joint portfolio tail loss conditional on the default of the FI less the joint portfolio tail loss conditional of the FI being solvent; or using the Myerson-Shapley value which in a network setting with a number (N) of FIs requires the incremental risk measure to be evaluated over all non-zero subgraphs of N.[[12]](#footnote-13) The principle is that capital surcharge or macro-prudential capital requirement for each FI should reflect the FI’s contribution to the joint portfolio loss. Chan-Lau (2010), for instance, specifies the systemic risk surcharge for a FI as the product of an incremental social loss function and the probability of default of the FI.

In those systemic risk measures that aim to estimate the joint portfolio loss distributions to reflect the co-movements in the cascade of losses for others from failure of a FI, the probability of default (PoD) of a FI and a matrix of pairwise conditional PoDs are needed. There are many methods to determine the PoD of an FI. Popular among them is to use CDS spreads directly to proxy PoDs, back them out from CDS spreads or use default estimates from data providers such as Moody’s KMV. The conditional pairwise PoD is obtained by (high at 95) quantile regressions of one FI’s PoD on that for another FI and common risk factors (Chan-Lau (2010)). The Goodhart and Sergoviano (2009) banking stability index is based on similarly constructed distress dependence matrix. They then use the probability of one or more FIs being in distress conditional on a particular FI being the trigger to determine the systemic risk impact of the FI. The major drawback of market price data based measures of systemic risk is that they can suffer from the paradox of financial stability (Borio and Drehman (2009)) or the paradox of volatility, issues first addressed by Hyman Minski (1982). These market data based statistical proxies for systemic financial fragility are at their lowest just before the point of great financial collapse and hence while there may be some information in the cross-sectional data of FI’s contributions to systemic risk, the aggregated systemic risk statistics are at best contemporaneous with the crisis in markets. [[13]](#footnote-14) While Adrian and Brunnermeir (2009) do address the paradox of financial stability[[14]](#footnote-15) problem by proposing a countercyclical forward Co-Var measure, this is not the case with most market based systemic risk measures.

Hence, to date there has been no comprehensive study of *TITF* and systemic risk measures based on the network topology of bilateral exposures and obligations in derivatives markets. Financial network models based on financial exposures are structural models that aim to depict causal chains of exposures and obligations of counterparties rather than rely solely on statistical correlations on market price based data for FIs. The only quantitative assessment to date of FIs’ overall derivatives liabilities and systemic risk impact is that of Segoviano and Singh (2010). The paper is based on the FDIC/OCC data on fair value of derivatives liabilities for FIs and the CDS spreads for these FIs as reference entities. The latter determines the conditional default probabilities and the so called distress dependence between FIs determines which FIs will fail conditional on failure of others. Segoviano and Singh (2010) find that the expected cumulative derivatives losses when cascaded in a series of insolvencies of top broker dealers exceed the capabilities of the Fed Reserve to provide backstops. This adds urgency for the need to conduct an in-depth structural analysis of the financial derivatives and the role of large FIs.  
   
In contrast to the Sergoviano-Singh framework where cascade effects are governed by the distress matrix based on the CDS spreads on the FIs, in the structural financial network model, the direction and magnitude of derivatives payables and receivables between FIs govern connectivity, concentration, network structure and system instability. The Markose *et*. *al.* (2010) network algorithm based on market shares of FIs and others in the gross notional of the derivatives contracts determines the ranking in the market for network links to and from each FI and hence also the bilateral links between agents. Based on 2009 FDIC and individually collected firm level data covering gross notional, gross positive (negative) fair value and the netted derivatives assets and liabilities for 202 financial firms which includes 20 SIFIs, the bilateral flows are empirically calibrated to reflect data based constraints on bilateral netting to best estimate counterparty risk (see, Sergoviano and Singh (2010)). This produces a tiered network with a distinct highly clustered central core of 16 SIFIs that account for 78% of all bilateral exposures and a large number of FIs on the periphery. A benchmark network model is established based on the aggregated data over all derivatives products for each FI. It is then stress tested in the classic Furfine (2003) way by an exogenous FI failure as the trigger and only direct losses from failure of counterparties is accounted for. Total losses from cascade effects can be estimated to see what proportion of total capital is wiped out. The topology of the network manifests the *TITF* phenomenon in that the failure of any member of the central tier will bring down other members with the contagion coming to an abrupt end when the ‘super-spreaders’ have demised. As these SIFIs account for the bulk of capital in the system – *ipso facto* no bank among the top tier can be allowed to fail highlighting the untenable implicit socialized guarantees needed for these markets to operate at their current levels.   
  
 **1.4 Structure of the Paper**

Section 2 will review the technical aspects of network theory and economics literature on financial networks. The main drawbacks of the pre 2007 economics literature on financial networks has been that models that are based on empirical bilateral data between counterparties were few in number to establish ‘stylized’ facts on network structures for the different classes of financial products ranging from contingent claims and derivatives, credit related interbank exposures and large value payment and settlement systems. Where bilateral data on financial exposures was not available, both empirical and theoretical models assumed network structures to be either uncorrelated and random (see, Nier *et. al.* (2007)) or complete networks (see, Upper and Worms (2004)). As will be argued, these approaches crucially do not have the *TITF* characteristics which will be modelled using a highly sparse core-periphery network structure. Only Craig and von Peter (2010) and Fricke and Lux (2012) who use empirical bilateral interbank data have highlighted the core-periphery network structure which has also been found by the author to be most pronounced in the bilateral data for derivatives liabilities in the Indian financial system.[[15]](#footnote-16) While the stability of financial networks has been usually investigated using the classic Furfine (2003) algorithm, sufficient emphasis has not been given to the way in which contagion propagates in highly tiered and clustered networks and stability of the system in terms of network characteristics has not been studied by economists. Following May (1972, 1974) and the recent approach of Wang *et. al.* (2003) and Giakkoupis *et.al.* (2005) which is increasingly being adopted in epidemiology and for internet security, the maximum eigenvalue for the network of derivatives liabilities expressed as a ratio of Tier 1 capital will be used to determine the stability of the system. Section 3 discusses the necessary network stability results using maximum eigenvalue and the corresponding eigenvector centrality of the FIs. Systemic risk index for each FI is based on its right eigenvector centrality and the super-spreader tax fund that can mitigate potential socialized losses from the failure of highly connected banks is designed. In contrast, the left eigenvector centrality index for the financial network will be called the vulnerability or exposure index. In the empirical Section 4, the empirical reconstruction of the derivatives network based on 2009 Q4 data is undertaken and the network analysis and a series of stress tests are conducted to investigate the consequences of the high concentration of activity of 16 or so SIFIs. The super-spreader tax is quantitatively derived and back tested to see if sufficient funds are escrowed from the super-spreader tax generated to stabilize the system. Section 5 concludes the paper and outlines future work.

**2.Financial Network Analysis**

Networks are defined by a pair of sets (N, E) which stand for nodes N={1,2,3,…..,n}, and E is a set of edges. In financial networks nodes stand for financial entities such as banks, other financial intermediaries and their non-financial customers. The edges or connective links represent contractual flows of liquidity and/or obligations to make payments and receive payments. Let i and j be two members of the set N. When a direct link originates with i and ends with j, viz. an out degree for i, we say that it represents payments for which i is the guarantor. Note, an agent’s out degrees corresponding to the number of its immediate neighbours or counterparties and is denoted by *ki.* A link from j to i yields an in degree for i and represents cash inflows or financial receivables for i from j.

**2.1 a Adjacency Matrix and Gross Flow Matrix For Derivatives**

Key to the network topology is the bilateral relations between agents and is given by the adjacency matrix. Denote the (N+1) x (N+1) adjacency matrix A = (aij)I , here I is the indicator function with aij=1 if there is a link between *i* and *j* and aij=0, if not. The Nth agent will be represented by the non-bank FIs such as Monolines, hedge funds and insurance companies. The N+1th agent represents participants not including the 202 US and European FIs. This is also used to balance the system. The set of agent *i*’s *ki* direct neighbours Ξi = {∀ j, j ≠ i, such that aij = 1} gives the list of i’s counterparties to whom *i* has to make payments or fulfil other financial obligations. The adjacency matrix becomes the gross flow matrix X such that xij represents the flow of gross financial obligations, GNFV, from the derivatives seller (the row FI) to the derivatives buyer j (the column FI). Note GNFV and GPFV is a fraction (typically 10 percent) of the gross notional for which the firm is a derivatives seller or buyer, respectively. The total gross payables in terms of GNFV for FI i is the sum over j columns or counterparties, Gi =  while the total gross receivables or total GPFV for each j is the sum taken across the i rows Bi = . This is shown below :

**X =** (1)

The zeros along the diagonal imply that FIs do not lend to themselves (see, Upper, 2007) or in this case of derivatives and in case of contagion, FIs do not ‘infect’ themselves. There can be asymmetry of entries such that for instance G1 need not equal B1. However, aggregate GNFV including that of the N+1 entity  will be made to balance with the 

**2.1.b Bilaterally netted Matrix of Payables and Receivables**

Consider a matrix M with entries (xij - xji) which give the netted position between banks i and j. For each FI i the positive (negative) entries, mij > 0 ( mij < 0), in row i give the net payables vis-à-vis FI j and the sum of positive (negative) entries for FI i is its total bilaterally netted payables (receivables) across counterparties. Note, FDIC Call Reports and individual Financial Reports for SIFIs give the firm level derivatives assets and liabilities data. Each of these is the appropriately signed (positive for derivatives assets and negative for derivatives liabilities) sum of bilaterally netted market value of derivatives payables and receivables by counterparty and product and also adjusted for collateral for each bank. This is reported in aggregate for *all* derivatives products rather than product by product. Hence, in the algorithm that constructs the bilaterally netted matrix M for derivatives aggregated over all products, we will impose the condition that, mij > 0 , will equal the data determined derivatives liabilities for the FI up to some margin of error. Likewise, the sum of the negative entries, mij < 0, for each FI i in the ith row gives its total bilaterally netted receivables, which as explained above will be calibrated to satisfy the data determined derivatives assets for that FI. Note the matrix M is a skew symmetric one with entries mij = - mji . To analyse the dynamics of the cascade of failures following the failure of the ith bank, the matrix that is relevant will only contain the positive elements of the M matrix and also the zero elements denoting no links exist or that there are exactly netted positions between (i,j). The causal direction of the contagion and hence systemic risk of a FI, follows from the failed FI, i, owing its counterparty j more than what j owes i. Further, as will be discussed in Section 4.2, the matrix transpose of Θ in (2) denoted as ****´ will be used, as it is customary for the net exposures of bank i to bank j relative to i’s initial capital, Ci0, to be greater than a threshold (signifying a proportion of i’s capital) before i is said to have failed. Hence, the matrix Θ that is crucial for the contagion analysis will have elements given as follows:



The matrix Θ is a non-negative real matrix as no element is negative and all elements are either zero or strictly positive real numbers, . In general, a matrix or vector will be said to be positive (non-negative) if all elements in it are positive (non-negative).

**2.2 Topology of Financial Networks: Complete, Random, Core-Periphery, Clustered and Small World**

Like many real world networks socio-economic, communication and information networks such as the www, financial networks are far from random and uncorrelated. In order to construct and characterize financial networks that have high concentration or localization of exposures with dense network interconnectivity between a few FIs, it is useful to briefly survey the well known classification of networks (see Table 2) and highlight the properties of what are referred to as small world networks[[16]](#footnote-17) which display properties that are a good fit for the financial network of exposures found in derivatives markets.

To incorporate the highly skewed connectivity of 26 broker-dealers and when compared to the others who are in the majority with few connections, a core-periphery structure (Borgatti and Everett, 1999) with small world network properties manifests it-self in the derivatives network. This has also been regarded to be a statistical signature of complex social systems (Watts,1998 and Watts and Strogatz,1999) namely, a top tier multi-hub of few agents who are highly connected among themselves (often called rich club dynamics) and to some other nodes in the periphery who show few if any connections amongst themselves. The properties of small world networks and how contagion propagates through them will be briefly contrasted with that for the Erdös -Renyi random graph and also the Barabási and Albert (1999) scale free networks. In order to construct a network for financial derivatives which shows dominance of few players with a 92% and upwards of concentration of derivatives exposures to one another, use will be made of network statistics and dynamics described below regarding high clustering, preferential attachment in link formation and identification of members with high ‘rich club’ statistics.

Networks are mainly characterized by the following network statistics – (a) the probability of any two randomly selected nodes in a network being connected is denoted by p with there being total of N(N-1) possible links for directed graphs and N(N-1)/2 for undirected graphs, (b) measure of local interconnectivity between nodes called clustering coefficient, (Δi denotes the clustering coefficient for node i and Δ is the clustering coefficient for the network) ; (c) The shortest path length of the network estimates the average shortest path between all pairs of randomly selected nodes; *and* (d) degree distribution which gives the probability distribution P(k) as a function of the degrees k of each of the nodes, and p(k) gives the probability that a randomly selected node as exactly k links. The average number links per node is given by <k> = ∑kk p(k) and the variance of links <k2> = ∑kk2 p(k). Where empirical sample data is used, p(k) = Nk/N-1 where Nk is the number of nodes with k links. Text book prototypes of regular, random and scale free networks have properties in respect to the network statistics given in Table 2. As will be explained, small world networks described by elements along the diagonal of Table 2 have one property in common with each of the other network types.

Table 2: Networks Statistics: Diagonal Elements Characterize *Small World* Networks

|  |  |  |  |
| --- | --- | --- | --- |
| **Properties**  **Networks** | **Clustering Coefficient** | **Average Path**  **Length** | **Degree**  **Distribution** |
| **Regular** | ***High*** | **High** | **Equal and fixed In/Out degrees to each node** |
| **Random** | **Low** | ***Low*** | **Exponential** |
| **Scale Free / Power Law** | **Low** | **Variable** | ***Fat Tail Distribution*** |

In Erdös-Renyi random networks in which connectivity between any two nodes is uncorrelated, the probability distribution of degrees is given by

P(k) = (3)

The exact solution to the last term above is achieved in the limit when N →∞. The Erdös-Renyi random networks show no very highly interconnected node and little local interconnectivity or clustering. They have short path lengths due to the random link formation. The average shortest path between any two arbitrarily chosen nodes is found to be “small” and bounded by the logarithm of the total number of nodes in the system.

Clustering in networks measures how interconnected each agent’s neighbours are and is considered to be the hallmark of social and species oriented networks. Specifically, there should be an increased probability that two of an agent’s neighbours are also neighbours of one another. For each agent with *ki* neighbours the total number of all possible directed links between them is given by *ki (ki-1)*. Let *Ei* denote the actual number of links between agent i’s *ki* neighbours, viz. those of i’s *ki*neighbours who are also neighbours. The clustering coefficient i for agent i is given by

Δ i =  [[17]](#footnote-18) and Δ = . (4)

The second term which gives the clustering coefficient of the network as a whole is the average of all Δi’s. Note that the clustering coefficient for an Erdös-Renyi random graph is Δrandom = p. This is because in a random graph the probability of node pairs being connected by edges are by definition independent, so there is no increase in the probability for two agents to be connected if they were neighbours of another agent than if they were not. A high clustering coefficient for the network corresponds to high local interconnectedness and of a number of agents who operate as hubs in the core. In regular networks, all nodes have the same number of links to and from them, and they show high and local clustering but do not have short path lengths. Scale free networks have highly skewed distribution of links that follows a power law in the tails of the degree distribution, that is the probability of a node possessing k degrees is given by

p(k) = , (5)

where  >0 is called the power law exponent. Hence, there are some nodes which are very highly connected and many that are not. To generate power law statistics for nodes either in terms of their size or the numbers of links to/from them, Barabási and Albert (B-A,1999) proposed a process called preferential attachment, whereby nodes acquire size or numbers of links in proportion to their existing size or connectivity. [[18]](#footnote-19) However, as noted by Zhou and Mondragon (2003) the growth dynamic of the B-A algorithm means that while highly connected ‘rich’ nodes get richer, it does not follow that they could themselves preferentially connect to one another. An important discovery that was made by Watts (1998) and Watts and Strogatz (1999) with regard to social networks is that while small world networks like scale free networks have in-egalitarian degree distribution with some very highly connected nodes, the central tiering of highly clustered nodes which work as hubs for the peripheral nodes (who have few direct connections to others in the periphery) is a signature feature only of small worlds. The hubs also facilitate short path lengths between two peripheral nodes. So unless there is a modification of the B-A algorithm that was suggested by Zhou and Mondragon (2003) in order that there is scope for preferential attachment or assortative mixing among the nodes with large number of outdegrees and not just a preference for high degree nodes (disassortative mixing ) by low degree nodes, the combined core-periphery structure with the high clustering of small world networks with the characteristics given in Table 1 will not occur. Hence, the premise behind the modified B-A algorithm will be adopted to get the extreme concentration of bilateral exposures between SIFIs in the derivatives markets.

Apart from the clustering coefficient, two further statistics will be used to characterize networks that show high concentration of activity and also the identification of network centrality of highly connected nodes. The first of this is the rich club coefficient. We will use the rich club coefficient, k) to identify highly connected nodes who form the club which is characterized by a fully connected network, Colizza *et.al.* (2006). The latter yields a coefficient of 1 and k# will denote the critical number of out-degrees the nodes need to have to be part of the largest sized rich club with k) =1. The rich club coefficient is estimated as:  
   
k) = . (6)

Here N>k refers to the number of nodes with degrees higher than a given value of k and E>k denotes the number of connected edges among the N>k nodes. The denominator divided by 2 gives the maximum number of possible edges in any direction as in an undirected graph.   
   
The network centrality measure that has been found by the author to correlate best with the capacity of a FI to cause the largest contagion losses on others in the Furfine (2003) type stress test is its eigenvector centrality statistic obtained from matrix in equation (2). The algorithm that determines it assigns relative centrality scores to all nodes in the network based on the principle that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes. Denoting as the right eigenvector centrality for the *ith* node for matrix , the centrality score is proportional to the sum of the centrality scores of all nodes to which it is connected (ie. kioutdegrees or direct neighbours of i). Hence,

=  (7)

For the centrality measure in (7), we take the largest eigenvalue, λmax , and its associated eigenvector. The *ith* component of this eigenvector then gives the centrality score of the *ith* node in the network. Using vector notation for this, we obtain the eigenvalue equation for the matrix in (2) for the eigen-pair (λmax, **)** as:

 **=** λmax . (8.a)

Note, for a non-negative matrix in (2) with real entries, λmax is a real positive number and the eigenvector  associated with the largest eigenvalue has non-negative components by the Perron-Frobenius theorem (see, Meyer (2000) Chapter 8). Positive values for the centralities of all nodes of matrix in (2) are guaranteed by Perron-Frobenius theorem only if in (2) is irreducible.[[19]](#footnote-20) For matrix  clearly, given equation (7), those nodes in the periphery with no out-degrees will have zero eigenvector centrality.

Finally, from the perspective of the measure of systemic risk the so called right eigenvector of  matrix given above in (8.a), as will be discussed, is what matters. A FI’s systemic risk index will be based on this. It measures the impact of FIs’ total liabilities given by the row sums of matrix in (2) on the stability of the system characterized by the maximum eigenvalue. The so called dual left eigenvector, on the other hand, gives the impact of the exposures of each FI to others and hence can be seen to yield vulnerability indices. The left eigenvector of denoted by **v1** is defined as

**v1** **= **´**v1** **=** λmax **v1**. (8.b)

Note both the left and right eigenvectors yield the same maximum eigenvalue for the matrix in (2).[[20]](#footnote-21)

**2.3 Economics Literature on Financial Networks**Pre 2007, financial network models in the economics literature have yielded mixed results. An influential and early work on connectivity in a financial network and that of financial contagion is that of Allen and Gale (2001). They gave rise to a mistaken view that follows only in the case of homogenous graphs,[[21]](#footnote-22) ie. increasing connectivity monotonically increases system stability in the context of diversification of counterparty risk. Battiston *et al.* (2010) have correctly shown that connectivity and stability of a network is not a simple monotonic relationship but that beyond a point, connectivity in a network could increase instability. However, Battiston et al. (2010) also do not offer insights into what sort of network topologies give rise to concerns about financial intermediaries being *TITF.*  Indeed, the derivatives based risk sharing networks, as discussed above, need to be characterized by a highly interconnected central tier composed of high degree hub nodes which has been called a rich club. In contrast, the bulk of numerically based studies in financial contagion work ( see, Nier *et al.* (2007) and Gai and Kapadia (2010)) has been confined to Erdös-Renyi random graphs and while they can be regarded as first steps, as financial networks are far from random, they have some way to go.

As little empirical work has been done to date on network structures of the specific markets underpinning off-balance bank activity such as CDS responsible for triggering and propagating the 2007 crisis, it must be noted that the bulk of the empirical financial network approach has been confined to interbank markets for their role in the spread of financial contagion (see, Furfine (2003) and Upper (2007)). However, the use of the entropy method[[22]](#footnote-23) (see, Upper and Worms (2004) and Boss *et al.* (2004)) for the construction of the matrix of bilateral obligations of banks which results in a complete and as homogenized network structure as possible, greatly vitiates the potential for network instability or contagion.

Recent work by Craig and von Peter (2010) using bilateral interbank data from German banks and also Fricke and Lux (2012) for the Italian interbank market have identified the tiered core-periphery structure and find that bilateral flow matrix (X) unlike in a complete network or as in a Erdös-Renyi random network is sparse in the following way:

X = . (9)

Here, CC stands for the financial flows among the core banks in the centre of the network, CP stands for those between core and periphery banks, PC between periphery and core banks and PP stand for flows between periphery banks. The sparseness of the matrix relates to the fact that PP flows are zero and banks in the periphery of the network do not interact with one another. The localized clustering in the central core resembles the small world network property of being *too interconnected* and corresponds to the data based CDS market network where this property was first identified in Markose *et al.* (2010). Hence, the criticism Craig and von Peter level at extant financial networks literature is worth stating here. They say that many interbank models proposed in the economics literature (e.g. Allen and Gale (2000), Freixas *et. al.* (2000), and Leitner (2005)) ignore the tiered structure and do not analyse it in any rigorous way : “the notion that banks build yet another layer of intermediation between themselves goes largely unnoticed in the banking literature”. Craig and von Peter (2010) find that the tiered character of this market is highly persistent which could coincide with an outcome of competitive co-evolution, Markose (2005), in that to retain status quo in market shares, the core banks are hugely geared to the arms race involved there (see, also Galbiati and Giansante (2010), Giansante (2009)). Craig and von Peter (2010) go on to note that “ *the persistence of this tiered structure poses a challenge to interbank theories that build on Diamond and Dybvig (1983). If unexpected liquidity shocks were the basis for interbank activity, should the observed linkages not be as random as the shocks? Should the observed network not change unpredictably every period? If this were the case, it would make little sense for central banks and regulatory authorities to run interbank simulations gauging future contagion risks. The stability of the observed interbank structure suggests otherwise*.” Fricke and Lux (2012) also report on the persistence of the central core structure in the overnight Italian interbank network of bilateral obligations which had about 28% of all banks before the 2007 global financial crisis and 23% surviving afterwards.[[23]](#footnote-24)

From the author’s experience of mapping the financial networks based on actual bilateral data of financial institutions for the Indian financial system, there appears to be a distinct variation in the core-periphery hierarchical structure noted by Craig and von Peter (2011) in the different types of financial activities. In their derivatives or contingent claims exposures and obligations, FIs show a far more marked concentration in the core both in terms of financial obligations and connectivity, with few banks in the core and a large number of them in the periphery. In non-contingent claims based borrowing and lending the interbank market shows more diffusion in the core with a larger number of banks in the core. The least hierarchical and sparse is the RTGS payment and settlements systems where there is a distinct lack of identifiable periphery banks in the Indian context. That the credit based interbank markets have different network properties to RTGS payment and settlement systems has also been noted by Kyriakopoulos *et. al.* (2010). [[24]](#footnote-25) Their findings on the network topology of the Austrian payment and settlement systems have been found to correspond to the study of the Fedwire payment and settlement system by Soramaki *et. al.* (2006). Bech and Atalay (2008) did a detailed study of the network topology of Fed Funds market and found that the clustering of the system was limited and that small banks lend more to big banks than to their own sized banks there was disassortative linking. They found that this disassortativity was reduced when links were weighted by value of flows. Hence, our emphasis on the need for empirical calibrations that reflect actual market concentration in activity or the use of full bilateral data on financial obligations between counterparties.

Finally, the presence of highly connected and contagion causing players typical of a clustered complex system network perspective is to be contrasted with what some economists regard to be an equilibrium network. Recently, Babus (2009) states that in “an equilibrium network the degree of systemic risk, defined as the probability that a contagion occurs conditional on one bank failing, is significantly reduced”. Indeed, the premise of *TITF* is that the failure of a big bank will increase the failure of another big bank, indicating that network formation in the real world are different from those assumed in economic equilibrium models.

**2.4 Eigenvalue Perspective of Network Stability**

The analysis and management of the stability of financial networks given in this paper from the perspective of maximum eigenvalue and eigenvector centrality has been influenced by the work of May(1972,1974), Wang *et. al.* (2003) and studies on the spread of epidemics in non-homogenous networks with hierarchies (see Kao, 2010, p62). In a seminal work based on random matrix theory, May (1972,1974) extended the Wigner condition of eigenvalues for complete matrices to sparse random networks. May was among the first to state that the stability of a dynamical network based system will depend on the size of the maximum eigenvalue of the weighted adjacency matrix of the network. Assuming the matrix entries are zero mean random variables, May (1974) derives a closed form solution for the maximum eigenvalue of the network in terms of three network parameters: p, the probability of connectivity, N the number of nodes and which is the standard deviation of node strength The May (1974) result states that network instability follows when the maximum eigenvalue is greater than one, viz.  > 1. There is a trade off between heterogeneity in node strength[[25]](#footnote-26) on the one hand, and connectivity and size of the network, on the other. Large networks with highly divergent node strengths have to be sparse with low connectivity to remain stable over time. Only homogenous networks with very low can increase connectivity and the number of nodes and remain sustainable. In networks with high variance to mean ratio in degrees and with tiered hierarchies of highly connected nodes where there is higher probability that any node is connected to a highly connected one, the direction of the epidemic which starts in a central hub follows a distinct hierarchical pattern with the highly connected nodes being infected first and the epidemic then cascading toward groups of nodes with smaller degrees, Kao (2010). Further, the epidemic dies out at great speed once the super-spreaders are eliminated, May and Lloyd (1970). In contrast, in uncorrelated random graphs, the epidemic lasts longer and also reaches more nodes. In terms of epidemic control, clustered networks enable targeting of specific individuals as opposed to inoculating the whole population in a random graph. Sinha (2005) and Sinha and Sinha (2006), also find that while both the small world and the Erdös- Renyi random graph show instability at the condition given by May (1974), the lack of structure in a random graph results in a worse capacity of the system to cope with the contagion.

In Markose  *et. al.* (2010, 2012) it was shown, for the first time in the case of financial networks, how an Erdös-Renyi random network equivalent to the empirically calibrated highly clustered core-periphery network for the CDS market propagates instability in a radically different way from the latter. The failure of highly connected units increase the probability of failure other similarly connected units, an aspect of the *TITF* phenomena. Structurally, as will be seen, the interconnected hubs can contain the liquidity shocks and prevent them from going to the extremities, but only if there are adequate buffers. This paper will operationalize the link between the stability condition on the maximum eigenvalue for the financial network and the corresponding eigenvector centrality parameter for FIs as the basis of their contribution to systemic risk.

**3. Contagion and Stability Analysis**

The study of the topology of networks in order to characterize their dynamical and stability properties has been actively studied especially in the context of ecology of species and in epidemiology. In financial network model the analysis of contagion from specific node failure has typically used the classic Furfine (2003) methodology. However, to date very little use has been made of network analysis in the monitoring and management of systemic risk. The first step here is to convert the well known Furfine (2003) algorithm to model a dynamical network system for population failure among FIs.

**3.1 Furfine (2003) Methodology : Cascades from Failure of a Trigger Bank**

We follow the round by round or sequential algorithm for simulating contagion that is now well known from Furfine (2003). Starting with a trigger bank i that fails at time 0, we denote the set of banks that fail at each round or iteration by Dq  q= 1,2, … . Note the superscript q denotes the qth iteration. The cascade of defaults occur in the following way:

(i) Assuming tear ups but no novation of derivatives contracts and zero recovery rate on the trigger bank i’s derivatives liabilities to bank j which exceed j’s liabilities to i become net losses to j. Bank j is assumed to have failed when j’s net losses from i when taken as a ratio of j’s Tier 1 capital (reported in the last column of Table A.1 in the Appendix) is greater than a threshold . That is,

( xij - xji )+/ Cj >  .

This threshold  signifies a percentage of bank capital which can be regarded as specifically being held to buffer losses in this sector of activities. This threshold is assumed to be the same for all banks. The experiments assume a 

(ii) A second order effect of contagion follows if there are some banks z, {z}, ∉**D**1 , ie. those that  
did not fail in round 1, suffers losses due to counterparty failure such that the net losses are greater than a proportion  of its capital:

[( xiz - xzi )+ +]/ Cz  > .  
The summation term aggregates the net loss of derivatives cover to z from all banks j, j ≠ i , which demised in the first iteration.

(iii)This then iterates to the qth round of defaults if there are some banks v, {v} ∉{D1 ∪ D2 …. ∪ Dq-1}, ie. has not failed till q-1, such that

[( xiv – xvi )+ +]/ Cv ≥ .

(iv)The contagion is assumed to have ended at the round q# when there are no more banks left or none of those that have survived fail at q#.

**3.2 Financial Network Stability Analysis**Using the matrix transpose ****´from (2) and following the epidemiology literature, the terms for bilaterally netted liabilities from j to i as a ratio of i’s Tier 1 capital at the initial date t=0, can be viewed as a pairwise (i,j) heterogeneous rate of infection which a failed FI j will inflict on i. Likewise, the threshold 0<represents the rate of cure and taken as a proportion of Tier 1 capital of FIs, it can be used as a buffer to offset losses. The threshold  is assumed to be homogeneous for all FIs.

In matrix notation the equation for the dynamics of the cascade of failures conditional on the failure of the trigger bank(s) at some initial date, t=0, is derived below. Consider the initial column vector **U0** with elements (u10 , u20, ..... uN0) which gives the probability of failure of FIs. **U0** = (u10 , u20, ..... uNt) = (1, u20, ..... uN0) indicates the trigger bank that fails at initial date 0 is bank 1, and the non-failed banks have a low initial failure rate of ui0 = /(1- ). Here, as can be taken to be the cure rate, (1-) is failure rate for any FI.[[26]](#footnote-27) Going forward, the ratio of capital at q to initial capital, Ciq/Ci0 , gives the probability of survival for a non-failed bank and hence at the q th iteration, the probability of failure of a non-failed bank is given by uiq = (1- Ciq/Ci0). In the course of the cascade at the q+1th iteration, the uiq+1 of non failed banks represents the probability of failure at q+1:

uiq+1 = (1- ) uiq + (10)

= (1- uiq+1 <1.

The first term in (10) is the product of the failure rate (1-and the probability of failure for the ith FI at q given by (1-Ciq/Ci0).[[27]](#footnote-28) The latter term goes to 1 as i’s capital is depleted by losses from failed banks. The second term in (10) sums up the infection rates sustained from its failed counterparties. Here, let ujq1=1 indicate those j ≠ i that failed at qth iteration and these will with probability 1‘infect’ all i non-failed counterparties in the edges (j,i) of matrix ****´ from (2) where i is exposed to j and there is zero impact between failed entities themselves (j,j). Non-failed banks, j, at q have zero impact on others j ≠ i and satisfy (10) if they survive to q+1. FIs that fail prior to q-s, s>1, have zero impact on both themselves and others.

**U**q +1 is a non-negative vector gives the incremental failure of banks at q+1 (viz. banks that have not failed previously) where an indicator function F(z) sets row uiq+1 in the vector **U**q +1 to equal uiq+11=1 to denote the failure of bank i, uiq+1 in (10) if i survives and uiq+10 = 0 if FI had failed before q. Thus,

uiq+1 = F(zF(z) = uiq+11 = 1 if z > 1,F(z) = uiq+1 if 0 <z < F(z) = uiq+10 =0 (11.a)  
 with z = + (1- uiq (11.b)  
 and z = 0 for all uiq-s1, s ≥ 1, viz. i failed on or before q. (11.c)

Hence, in matrix notation the dynamics of the cascade is

**Uq +1** = **[**´ + (1 - **** ]**Uq = QUq** .(12)

Here, ****´ is the transpose of the matrix in (2) with each element ij´= jiand **I** is the identity matrix. Recall the elements of the 2nd row of ****´ take the following form with for example, positive entries in (13) for counterparties 1, 3 and N and with to indicate that an FI does not ‘infect’ itself:

****2´ = . (13)

Thus, for example with trigger bank that fails at the initial date being bank 1, from (10) and (11) and noting that ui0 = /(1- ), bank 2 will fail at the first iteration q=1 if + That is, initial capital C20 must exceed the buffer  C20and losses from failed counterparty at q=0 for bank 2 (or any other non-failed bank) to survive to q=1. FI effectively fails at q>1 from (11.b) when the sum of the rates of infection from others plus its own failure rate (1- uiq exceeds 1. 

The total number of failed FIs at q is given by:

N#q  = ∑q ∑i uiq1 . (14)

For some finite q, N#q = N, ie. the whole population can be wiped out. For this at steady state denoted by , will be a trivial vector of zeros and if some nodes are to survive then is non-negative vector with some ui# <1. The system stability of (12) will be evaluated on the basis of the power iteration of the initial matrix **Q=[**´ + (1 - **** ]. From (12), Uq takes the form :

**U**q= [****´ + (1-)**I]**q**U0**  = **Q**q **U0** .  (15)

It can be shown that the stability of the system is governed by the maximum eigenvalue of the initial matrix **Q=**[****´ + (1 - **** ] when it satisfies the condition :

max (**Q) <** 1 . (16)

Expressing the eigenvalue equation in matrix notation, the qth power of matrix **Q** in (15) is:   
   
 **Q****V = DV** → **Q** q**V = D**q**V** (17)

Here **D** is a diagonal matrix with the eigenvalues (1 ,2, ...., N) of **Q** arranged along the diagonal from the highest to the lowest with 1 = max ≤ 2 ≤ .... ≤ N and the NxN matrix **V= [v1 v2….. vN]** is an array of columns of eigenvectors.[[28]](#footnote-29) As the set of eigenvectors in **V** can span the N dimensional vector space of real numbers **RN**, the vector **U0 ∈ RN** can be expressed as a linear combination of the vectors in **V**. Then the qth power for **Q** q **U0** in (15) with **g** being anx1 vector of weights is as follows:

** Q** q **U0** = **Q** q**Vg** **=** ***.*** (18)

Factoring out, in (18) such that

**Q** q **U0 =** ( + **+ .... + )** (19)

Limit q →∞ **Q** q **U0 =**  , if max > 2.

Firstly, if max > 2, then the stability of (19) is governed by the first term viz. the maximum eigenvalue and its corresponding eigenvector. Further, for the ‘infection’ from failed banks to die away and **Uq** tends to zero, a necessary condition is max <1.

Using the Shift-Eigenvalue theorem, given the eigen-pair (1(****´), **v1** ) for the matrix ****´ in (8) where **v1** is a left eigenvector of matrix **** in(2), then as (1-) is a constant in (12), ((1(****´)+ (1-)), **v1**) is the eigen-pair for the matrix **Q =** [****´ + (1-)**I]**. Hence, the stability condition is:

1 (****´)+ (1-) < 1 → 1(****´)= max(****´) <  and max(****´)> 0.[[29]](#footnote-30)



max(****´)> 0, so by the Perron-Frobenius Theorem vector **v1** is non-negative and likewise for the right eigen-vector in (8.a). The nodes with high eigenvector centrality in vector can be identified as those with the greatest spreading power. Clearly, targeting such nodes and ‘inoculating’ them to reduce the instability of the system by reducing max is becoming a prominent aspect of security design in networked systems (see, Giakkoupis *et. al.* (2005)).[[30]](#footnote-31)

In the management of contagion and the design of the super-spreader tax or inoculation measures, it is important to understand the relationship between the maximum eigenvalue of ****and its corresponding eigenvectors and **v1**. In what follows, the role of row sums in the stability of the system ****´ will be exploited. Firstly, it should be noted that the upper bound of the maximum eigenvalue max(****´) is given by maximum of row sums of the matrix ****´:

max ≤ || ****´||∞ = = maxi Si . (21)

Here, || . ||∞ infinity norm of a matrix is the maximum of row sums Si where Si = . Hence, high connectivity to large number of counterparties and also large exposures relative to capital contribute to the high row sums for FIs and with the largest of these being the upper bound of max. Further, from the von Mises power iteration algorithm, Ralston (1965), which simultaneously generates the dominant eigenvalue of ****´and its associated eigenvector, **v1**, the role of the scalar upper bound of || ****´||∞ can be seen as follows. Starting with an initial column vector **b**0 of 1’s, the power iteration of the matrix ****´ which yields the dominant eigenvector and eigenvalue is given by:

**bq+1 = =** . (22)

Here every element of the vector ****´q+1b0 is normalized by its own maximum row sum resulting in the highest eigenvector central element to be 1. The iteration is said to have converged at q+1 when  ****´**v1** **= bq+1 = bq** with an epsilon margin of error. Clearly, then at q+1, the scalar ||****´q+1b0||∞ = max while ||****´b0||∞ = || ****´||∞ = maxi Si. Normalizing by the maximum row sum in (22) shows the direct relationship between Si = and max as in the result given in (21). However, for purposes of designing the super-spreader tax, normalizing the numerator in (22) by the Euclidean norm ||****´bq||2 will be followed so that the scalar product **v1**´**v1=** =1. Note, the method of normalizing does not alter the converged values for the max for a matrix while the corresponding eigenvectors using the two methods differ by a constant multiplicative factor. [[31]](#footnote-32) Finally, the power iteration algorithm delivers the two ingredients needed to design systemic risk measures and mitigants for an interconnected system in terms of max and also the precise ranking of the centrality measures of the individual FIs.

**3.3 Mitigation and Management of Financial Contagion: Super-spreader Tax**

Most financial networks leading to contagion can be characterized by the system in (12) and they are prone to instability at max(****´)>  The latter is the prefunding threshold rate which is typically the same for all FIs. While max which governs stability is identical for the matrix **** in (2) or its transpose ****´, the right and the left eigenvectors, denoted respectively by  and **v1,** are different. In directed financial networks, those who are central in terms of the right eigenvector instigate systemic risk through excessive liabilities and can inflict losses on those who have high exposures to them. Those who are central with respect to the left eigenvector, viz. high on the vulnerability index, then become potent propagators only if they have high correlation in their right and left eigenvector centralities. This being the case, the individual systemic risk centrality measures from the right eigenvector is recommended. Nevertheless, as the analysis of contagion in financial systems encapsulated in (12) and (15) suggests, the application of deterrence must be along the rows of ****´.

There are 4 ways in which stability of the financial network can be achieved:   
(i) Constrain the bilateral exposure of financial intermediaries;   
(ii) Ad hocly increase the threshold in (12) identically for all FIs;  
(iii) Change the topology of the network;   
(iv) Levy a capital surcharge commensurate to the right eigenvector centrality of a FI in (12).   
  
The first two measures do not price in the negative externalities and systemic risk associated with failure of highly network central nodes. Network topologies emerge endogenously and are hard to manipulate exogenously.

The aim of the super-spreader tax is to have FIs with high eigenvector centrality parameters to internalize the costs that they inflict on others by their failure and to mitigate their impact on the system by reducing their contribution to network instability as given by max. As discussed, critical to the von-Mises power iteration algorithm for the calculation of λmax are the row sums Si of the ith row in ´,

*Si*= . (23)

We create a new row sum *Si#*, for each node so that a super-spreader tax denoted by  is applied on the ith node in proportion to the right eigenvector centrality which is a measure of its systemic risk:

*Si#* = (24)[[32]](#footnote-33)

The rationale behind this application of the right eigenvector centrality of a node as the basis of the super-spreader tax is to enable a node to provide a buffer proportional to its own capacity to propagate contagion. Note, the funds that will be escrowed here using is not calibrated to mitigate failure of FI i due to its exposures to j counterparties, but it is in keeping with its own spreading powers.

Thus,

*Si#* < *Si*  for  > 0. (25)

We will consider two formulations of the super-spreader tax:  
   
  *=  , 0 <  ≤ 1 or  >1* . (26.a)

*=  , 0 <  ≤ 1 or  >1* . (26.b)

In (26.a), the super-spreader tax is a linear function of a FI’s eigenvector centrality. In (26.b) the super-spreader tax is set proportionate to rather than to . The advantage of this is that is a naturally normalized variable with . Further, with the super-spreader tax being a function of rather than , this will penalize nodes with higher eigenvector centrality more than others. This is useful when the dominant eigenvector central node is distinctly more dominant than others and hence the progressivity of the tax rate becomes more apparent with the application of than of . In contrast, when several nodes are equally dominant, the use of  *= * is more appropriate. Finally, as *0* *≤ <1*, equation (26.a) can extract more tax across the population with non-zero centrality measures than a tax rate using and hence also deliver faster rates of decline of the max of the system.   
  
The new matrix associated with *Si#()* will be denoted as ´#(). The alpha parameter when set at 0 obtains the max associated with the untaxed initial matrix ´. When =1, each node is exactly penalized by or which yields the max for ´#(). Considering, 1> ,there is a monotonic reduction in the max associated with the matrices {´#()} corresponding to the monotonic reduction in row sums *Si#(1)* < *Si#(=1)* < ..< *Si#(=0.75)<...* < *Si#(=0.5)* < ...< *Si(=0)*. The case of 1 may apply when the initial matrix ´ needs a more aggressive application of the tax to stabilize the matrix to the point where *Si#(1)* for all i such that max<  for ´#(). Clearly, the size of , in particular if  >1 is needed to stabilize the system, the sustainability of such a market for risk sharing is in question. In fact, full stabilization to levels of max(´#) ≈ may not be technically possible and/or economically feasible. However, what the eigen-pair method of estimating and managing systemic risk in interconnected financial network can provide that certainly the market based methods cannot do is a mathematical benchmark for critical points.

The nature of systemic risk stabilization super-spreader fund is that it operates like an escrow fund. The funds commensurate to each FI’s surcharge as a proportion to their Tier 1 capitalis collected at some initial point whether within a CCP context or by the regulatory authority and the funds are deployed at the time of the failure of a FI for the collective good. The funds escrowed from the failing FI will contribute according to its right eigenvector centrality to restore to the others the domino losses delineated in the Furfine algorithm of Section 3.1. Note, the tax collected from non-failed banks themselves will also have to be used. The degree of stabilization being sought can be calibrated. A full on stabilization of matrix ´ to achieve max< may be not be necessary and may be prohibitively expensive as it requires the super-spreader fund to have collected the loss of capital for the system as whole at the end point q# of the Furfine contagion algorithm. The systemic risk super-spreader fund can be designed to work in a clustered hierarchical network where contagion takes a specific pathway amongst the central tier if a highly connected node fails. This *lite* super-spreader tax fund aims to cover the losses that the most connected nodes will inflict on their direct ‘big’ neighbours in the first tier, q=1. The empirical section will demonstrate these points quantitatively.

**4. Empirical Results: Network Analysis of the Calibrated Aggregated Global Derivatives Market**

We start with the construction of the network of bilateral financial obligations based on firm level data aggregated over all derivatives products and all over counterparties. The sample data for top 22 FIs of the 202 market participants is reported in Appendix Table A.1. The variables reported there is for derivatives gross notional, gross positive fair value, gross negative fair value, derivatives assets, derivatives liabilities and Tier 1 capital. As discussed in Section 2.3, the network algorithm works on the basis of the Zhou and Mondragon (2003) modified preferential attachment principle in order to achieve the sparse weighted adjacency matrix and the core-periphery network structure in order to reflect empirical observations on the high level of concentration of exposures among the top 12 derivatives broker dealers (see footnote 9). A further constraint that is met is that the bilaterally netted matrix crucial for the stability and contagion analysis, described in Section 3, satisfies the empirical data for each FI in regard to their derivatives assets and derivatives liabilities. We design a ‘shuffle algorithm’ for the bilateral netted M matrix described in Section 2.1b so that the errors are brought to about 2% of the actual data for the M matrix.

**4.1 Empirical (Small World) Core-Periphery Network Algorithm**

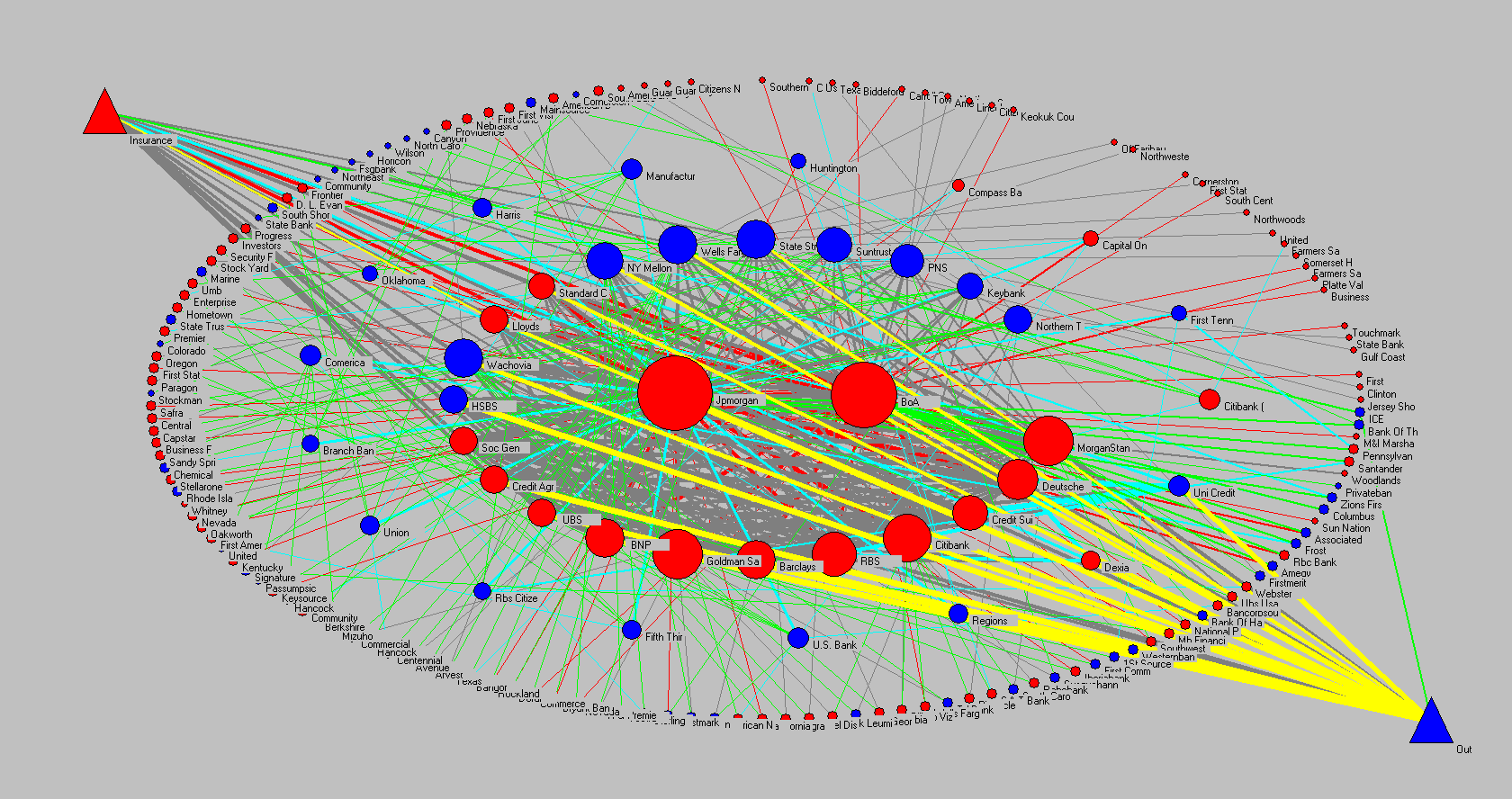
The adjacency matrix for X in (1) is constructed from the empirically based market shares in the relevant market data given in Appendix Table A.1. Our algorithm assigns out degrees for a FI in terms of its respective market share in terms of the GNFV of derivatives. Thus, JP Morgan with a 15% share will approximately have 31 out degrees. Due to the regional aspect of the global derivatives market, there is some regional clustering in FI’s allocation of out-degrees. Denoting:

: market share of derivatives gross notional   
 : Gross Negative Fair Value for which is a guarantor vis-à-vis its counterparties   
 : Gross Positive Fair Value for which is beneficiary vis-à-vis its counterparties

The algorithm allocates to each of row bank i’s counterparties j, a value of its GNFV equal to and if  , then bank i allocates the remaining to the external non-US and European financial entities which are collectively denoted as the N+1 agent. The column sums of matrix X in (1) are made to satisfy the GPFVj or Bj for each bank, and if , the remaining is bought from the external N+1entity. The gross flow X matrix so constructed using the above algorithm is a sparse matrix with a very high concentration of activity in the top left hand side of the matrix. The bilaterally netted matrix **M** of Section 2.1b is constructed to satisfy the condition that, mij > 0, will equal the data determined derivatives liabilities for the FI up to some margin of error. Likewise, the sum of the negative entries, mij < 0, for each FI i in the ith row gives its bilaterally netted receivables for all derivatives products, which as explained above is calibrated to satisfy the data determined derivatives assets for that FI. The network for the bilaterally netted matrix is plotted below.

In Figure 4 above red nodes denote net derivatives sellers and blue nodes are net derivatives buyers. JP Morgan and Bank of America occupy central status with a clear 2 tiers of FIs around them. Over 80% of the banks are in the periphery with almost no connectivity among themselves reflecting a very sparse adjacency matrix described in equation (9).

**Figure 4 Empirically Constructed Global Derivatives Network (Bilaterally Netted) Aggregated over all Derivatives Products for FIs and Outside Entity (Triangle): Empirical Small World Network in Tiered Layout**



The tiered layout in Figure 4 is constructed in the following way. First the full range of connectivity of all banks is taken in terms of the ratio of each bank’s total in and out degrees divided by total number of connected links of the most connected FI. FIs that are a ranked in the top 10 percentile of this ratio constitute the inner core. This is followed by a mid core between 90 and 70 percentile and a 3rd tier between 40 and 70 percentile. Those with connectivity ratio less than 40% are categorized as the periphery. The tiering algorithm that produced Figure 4 has the constraint that those in the periphery are not connected to one another.

The links from a node are out degrees and depict derivatives payables. The links are weighted and thicker the links, the larger the size of their obligations. The links are colour coded. The triangles in Figure 4 represent non-banks and non-US and European FIs.

**4.2 Global Derivatives Network Statistics (2009 Q4)**

As with any complex network it is important to understand the topology and structure of the network in terms of the network statistics based on the degree distribution.

Table 3 gives the network statistics for the empirically constructed global derivatives networks for the case for 202 nodes which involve banks only and 204 nodes where the last two nodes include non-banks and the external balancing node representing banks outside of US and Europe. As we do not have explicit data on these, they are modelled not to fail in the contagion experiments. Network statistics are reported for both these cases. Table 3 indicates the sparseness of the network which can be seen from the average number of in and out degrees of about 2.3 and low connectivity of only 1%. With no isolated subgraphs for the network, such a low level of connectivity manifests the very high efficiency of link formation. The distribution of out degrees from FIs shows greater asymmetry than does that of in degrees in terms of all higher moments given by standard deviation, skewness and kurtosis. Three statistics for the empirical highly tiered derivatives network stand out in contrast to an equivalent random graph. Firstly, the cluster coefficient is about 0.48 whereas that for an Erdös-Renyi random graph this will be closer to the connectivity for the network of 0.01. Also, random graphs have substantially low variance to mean ratios, <k2>/<k>, of less than 1 when compared to that of the empirically calibrated network for derivatives exposures of FIs. The highly asymmetric nature of the empirical CDS network is manifested in the large kurtosis or fat tails in degree distribution which is characterized by a few (two banks in this case) which have a relatively large number of in and out degrees (over 40) while many have only a few (as little as 1). One of the stylized facts emerging on financial networks where power law exponents, defined in equation (5), are estimated for the degree distribution is that the power law exponent for the financial flows implies fatter tails than those obtained for the degrees. In the 204 node case, the degree distribution tail exponent, ie. for the unweighted matrix, is about 2.1 and a similar number is obtained in the 202 node case of 2.15. In contrast, the weighted network shows a power law exponent of 2.49 in the 204 node case and a substantially fat tail in the 202 case with a power law exponent of 3.38.

**Table 3 Network Statistics for Degree Distribution for Derivatives Network 2009 Q4 (Bilaterally Netted Matrix)**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Initial Network Statistics (M+) Unweighted 2009 Q4** | **Mean Degree** | **Standard Deviation (σ)** | **Skewness** | **Kurtosis**  **#(Power Law exponent z**  **unweighted;weighted)** | **Connecti-vity** | **Clustering Coefficient** | **<k2>/<k>: Variance to Mean Ratio** |
| **204 Nodes Edges: 470**  **In Degrees  Out Degrees** | 2.30 | 4.24 | 3.64 | 15.58 (2.1\*; 2.49\*\*) | 0.011 | 0.483 | 7.8 |
| 2.30 | 4.95 | 4.76 | 28.45 | 10.65 |
| **202 Nodes Edges: 424**  **In Degrees** | 2.09 | 383 | 3.92 | 18.87  (2.15;3.38) | 0.0104 | 0.48 | 7.017 |
| **Out Degrees** | 2.09 | 4.60 | 5.29 | 35.02 | 10.12 |

# Power Law exponent is given in equation (5) with the first number (marked with \*) being the unweighted statistic and the second number (\*\*) is the power exponent for the weighted case.

Membership of the rich club given in equation (5) with φ(k)=1 is determined using out degrees, ki. Note the number of FIs with ki>0 is only 150 out of a total of 202 nodes. As Table 4 shows there are 4 distinct groupings of rich club statistics. The members included in the rich club statistic from φ(k)=1 to φ(k)= 0.91, are those who are part of an almost complete central cluster. These include 14 major broker-dealers. In the next section, it will be shown that threats can arise from those FIs that occupy the second set of rich club coefficients in the range {0.71 0.69} and also from a couple in the range {0.64, 0.69}. Recall, even though members with low rich club coefficients have fewer out degrees, those that connect to the inner core of the network will have high eigenvector centrality and hence the capacity to trigger contagion. The 100 or so FIs on the periphery have a rich club coefficient of 0.03.

**Table 4: Rich Club Statistics (see, equation (6))**

|  |  |  |  |
| --- | --- | --- | --- |
| **Out Degree**  **(k)** | **Rich Club Coefficient** | **Number of Nodes(N(k))** | **Selected Members** |
| **k>0** | **0.03** | **150** |  |
| **k>1** | **0.22** | **42** |  |
| **k>2** | **0.38** | **30** |  |
| **k>3** | **0.64** | **21** | Standard Charter |
| **k>4** | **0.67** | **20** |  |
| **k>6** | **0.69** | **19** | Societe Generale |
| **k>7** | **0.71** | **18** | Dexia, Uni Credit |
| **k>8** | **0.91** | **13** | New York Mellon, UBS,HSBC |
| **k>9** | **0.95** | **11** | Credit Suisse, Credit Agricole, |
| **k>10** | **1** | **9** | Barclays, BNP Paribas, RBS |
| **k>12** | **1** | **6** | Morgan Stanley |
| **k>13** | **1** | **4** | Deutsche Bank, Citigroup |
| **k>16** | **1** | **3** | Goldman Sachs |
| **k>17** | **1** | **2** | Bank of America |
| **k>30** | **1** | **1** | JPMorgan |

4**.2 Eigenvector Centrality and Furfine Stress Test Results**

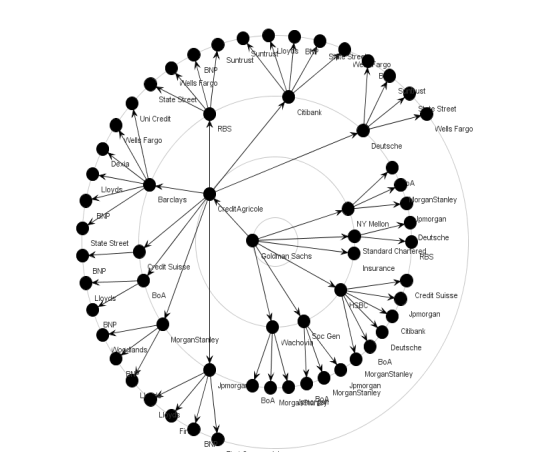
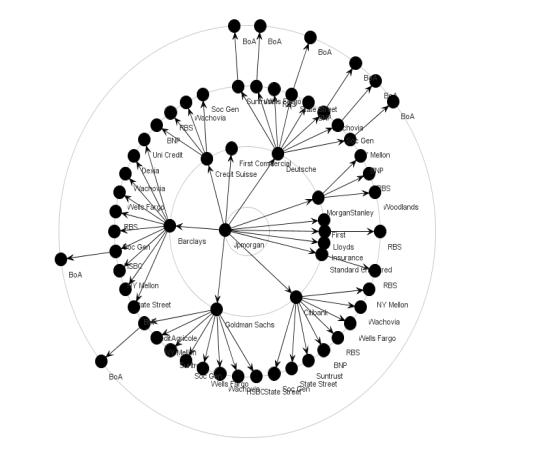
Here we will investigate the role of ‘super spreaders’ of contagion in the derivatives markets in terms of their network connectivity and their eigenvector centrality. The results of the Furfine contagion stress tests described in Section 3.1are also reported. Table 5 indicates that there is considerable difference in the ranking of FIs in respect to their systemic risk index and their vulnerability index (see equations 8.a and 8.b). Those that pose a major systemic risk threat as measured by the right eigenvector of matrix  are first Barclays with an index of 0.431, followed by JP Morgan and HSBC at around the 0.39 mark and then 5 US and European banks in the 0.25 range. What is interesting is that with the exception of JP Morgan and to a lesser extent Bank of America, the European banks, more than the US ones, rank highest with regard to the systemic risk index in 2009 Q4. HSBC Group and then JP Morgan bring about the largest, $39.73 bn and $32.38 bn, respectively, first round (q=1) losses of Tier 1 Capital in the Furfine contagion exercise. These amounts lost also form a large percentage of upto 69% of Tier 1 capital of the failed FIs in that round. JP Morgan, Barclays, BNP Paribas, Credit Agricole and Bank of America as trigger banks can lead to the demise of a maximum of 12 FIs to upto 7 FIs in the first round of contagion. In the exposure stakes measured by the vulnerability index, Goldman Sachs followed by Morgan Stanley rank the highest. Dexia, Uni-credit and Standard Charter are the least connected with less than 2% of total links. As shown in the last two columns of Table 5, in such densely clustered structures, there is high correlation between those who can inflict contagion directly on their counterparties and those who are directly vulnerable to the failure of others.

**Table 5: 2009 Q4 Derivatives Network Eigenvector Centrality and Furfine first Round Contagion Results for Top 20 FIs (ranked by weighted Systemic Risk Index given by Right Eigenvector Centrality,** see equations 8.a and 8.b**)**

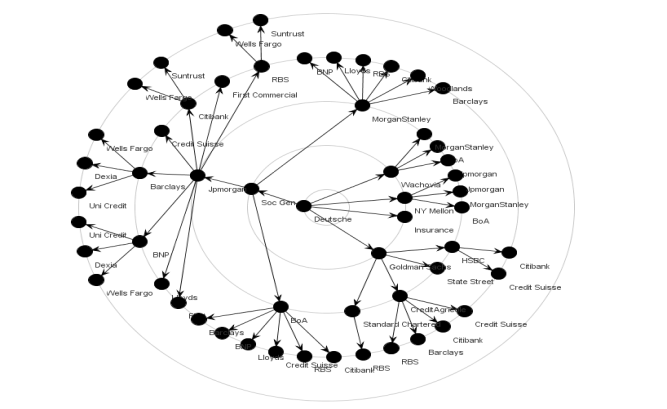
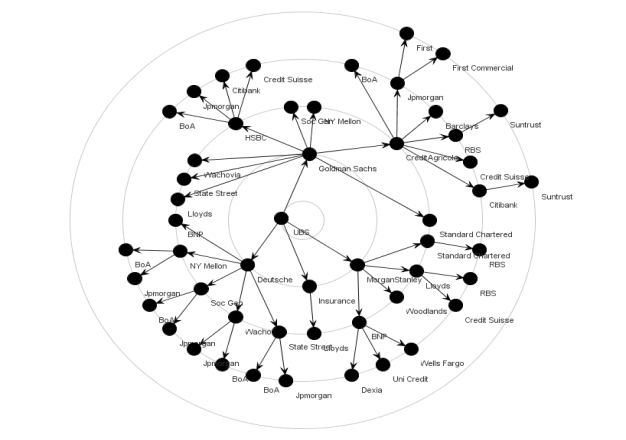
|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Bank Names (Trigger)** | **Right Eigenvector Centrality**  **Systemic Risk Index** | **Left Eigenvector Centrality**  **Vulnerability Index** | Contagion $ loss at q=1; 6% threshold (%) | Contagion of Trigger:  FIs Failed q=1 | Conditional failure of FI when others fail q=1 |  |
| **Barclays** | 0.431 | 0.076 | 20.42(40.96) | 8 | 8 |  |
| **JPMorgan** | 0.391 | 0.156 | 32.38(69.78) | 12 | 10 |  |
| **HSBC Group** | 0.388 | 0.146 | 39.73(66.17) | 4 | 6 |  |
| **Credit Agricole** | 0.259 | 0.089 | 9.68(42.44) | 8 | 5 |  |
| **BNP Paribas** | 0.255 | 0.072 | 7.16(28.07) | 9 | 9 |  |
| **Bank of America** | 0.253 | 0.137 | 12.08(43.19) | 7 | 6 |  |
| **Credit Suisse** | 0.244 | 0.183 | 11.28(33.01) | 4 | 4 |  |
| **Deutsche Bank** | 0.244 | 0.287 | 13.93(27.01) | 5 | 4 |  |
| **UBS** | 0.194 | 0.019 | 5.00(27.31) | 4 | 3 |  |
| **Uni Credit** | 0.192 | 0.025 | 15.88(25.97) | 3 | 2 |  |
| **Citibank** | 0.161 | 0.090 | 8.89 (24.34) | 5 | 5 |  |
| **Societe General** | 0.157 | 0.316 | 16.10(31.40) | 3 | 3 |  |
| **Goldman Sachs** | 0.155 | 0.559 | 3.97(20.20) | 7 | 6 |  |
| **Wachovia** | 0.133 | 0.222 | 9.82(25.12) | 3 | 3 |  |
| **Lloyds** | 0.132 | 0.143 | 9.29(19.28) | 3 | 3 |  |
| **Morgan Stanley** | 0.132 | 0.381 | 5.72(16.77) | 4 | 4 |  |
| **Dexia** | 0.061 | 0.022 | 5.23(10.75) | 2 | 1 |  |
| **Standard Chartered** | 0.051 | 0.313 | 3.93(8.24) | 2 | 1 |  |
| **RBS** | 0.049 | 0.069 | 0.37 (5.70) | 4 | 1 |  |
| **New York Mellon** | 0.048 | 0.191 | 0(0) | 0 | 0 |  |

Figures 5.a -5.f show the devastating consequences of the *too interconnected to fail* phenomenon. The empirically calibrated network of derivatives liabilities that has produced a highly clustered central core, results in a characteristic propagation of contagion. To date, this has not be sufficiently quantified or modeled in the case of financial networks. Note a 6% capital threshold has been assumed for the system dynamics in equation (12). As can be seen in Figures 5.a-5.f, whether the trigger bank directly kills 12 in the first round of contagion in the case of JP Morgan, 7 in the case of Goldman Sachs, 4 in the case of UBS or even only 2 in the case of Standard Charter, the contagion with no interventions can spread at great speed to the 3rd and 4th iteration (q=3, q=4 in the Furfine terminology given in Section 3.1 and shown as concentric circles in Figure 5) of the tiered OTC derivatives market. This will bring down a similar number (25) of FIs and the potential total loss of Tier 1 capital from *all* of these cases as shown in Figures 5.a-f is approximately $350 billion which is 25% of total Tier 1 capital of $1.413 trillion for the 202 FIs given in Table 1. Clearly, the failure of JP Morgan will trigger the fastest self-annihilating contagion which ends at q=2. This phenomena where a clustered group of banks stand or fail together is what we called *TITF*. Even though the overall connectivity of the global derivatives network was low at 1%, the tiered structure with heavy concentration of exposures is what causes this outcome. There is also the feature that others endowed with fewer outdegrees can provoke a slower growing contagion as in the case of Standard Charter with only 5 outdegrees. Finally, while there is some regional bias in that the failure of UBS and Deutsche Bank will certainly affect other big regional FIs, the global implications of failure are immediately apparent in the plots of the Furfine contagion algorithm.

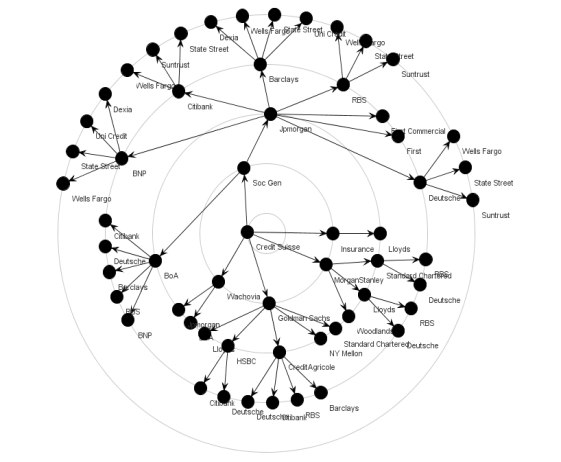
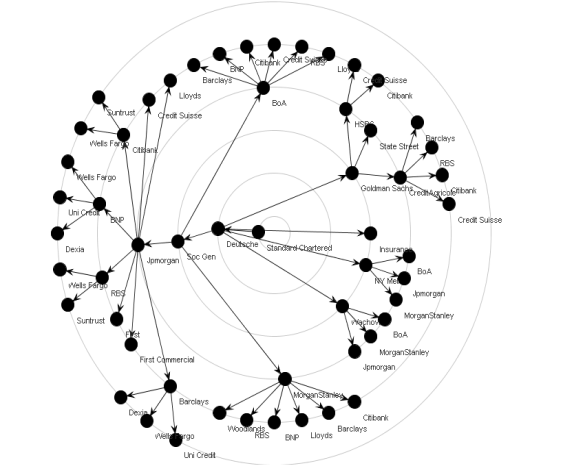
**Figure 5 Furfine contagion stress test on Empirical Calibrated Derivatives Network** 2009 Q4 (203 FIs) 6% Capital Threshold (Black nodes are failed banks; trigger bank in centre)

****

**Figure 5.a**Trigger bank JP Morgan: q=1 FIs failed 12 **Figure 5.b** Trigger bank Goldman Sachs: q=1 FIs failed 7  
Total FIs failed 25; 24.48% Tier 1 Capital Loss $345.94 Total FIs failed 25; 25% Tier 1 Capital Loss $355.75

****

**Figure 5.c** Trigger bank UBS: q=1 FIs failed 4 **Figure 5.d** Trigger bank Deutsche Bank : q=1 FIs failed 5   
Total FIs failed 26; 24.67% Tier 1 Capital Loss$348.66 Total FIs failed 25; 25% Tier 1 Capital Loss $355.75

**** ****

**Figure 5.e** Trigger bank Credit Suisse: q=1 FIs failed 5 **Figure 5.f** Trigger bank Standard Chartered: q=1 FI failed 1   
Total FI failed 25; 24.67% Tier 1 Capital $348.61 Total FI failed 25; 25% Tier 1 Capital $351.81

**4.4 Quantification and evaluation of the Super-spreader tax (2009Q4)**  
At a maximum eigenvalue for the system in (12)max (**Q)** 1.57 (or max(= 0.57 which is greater than = 0.06), the derivatives market is deemed unstable and the losses to the system as a whole from the failure of any of the top 20 eigenvector dominant banks, can be nothing short of catastrophic. The systemic risk is assessed solely on the basis of the  matrix of the ratio of derivatives liabilities to Tier 1 capital with the contagion dynamics governed by the equation (12). The total loss of Tier 1 capital from contagion is in the order of about $350 bn irrespective of which of the top FIs were to fail, with only the speed of contagion differing with different trigger banks. This is important to understand as the bulk of macro-prudential discussions focus on systemically important FIs rather than on unstable networks of financial liabilities. The objective of levying a derivatives market surcharge is that the socialized losses that arise from the interconnectivity of a FI have to be internalized by the FIs themselves.

In this section, we will evaluate the super-spreader tax based on a theoretical derivation in Section 3.3 and equations (26.a and 26.b). Note the eigenvector centrality for the top 20 banks for the initial untaxed system is given in Table 5. A surcharge on a FI’s capital commensurate to its right eigenvector centrality using the formulae in equation (26.a)  *= *(or equation (26.b) *= * ) is applied to the rows of ´ for different values of *0 <  ≤ 1*. This results in a monotonic reduction in the maximum eigenvalue  **#max** associated with the matrices {´#()} corresponding to the monotonic reduction in row sums *Si#(=1)* < ..< *Si#(=0.75)<...* < *Si#(=0.5)* < ...< *Si(=0)*. This is given in Figure 6.  
  
**Figure 6 Maximum Eigenvalue (#max ,Y-Axis) using different values of**  **> 0(Equation 26.a,b) for the Sequence of Matrices** {´#()} **: Note the initial max(**Q) **= 1.57, max(**)= 0.57 (latter plotted)

What is clear from Figure 6 is that the application of the super-spreader tax formula *= * in (26.a), rather than the one in (26.b), brings down the **#**maxof the system much faster for all values of **. At **when the **#**max (´) of the system comes close to =0.06, the maximum eigen-value algorithm ceases to converge. There are clearly technical and economic limits to how much a given unstable system can be stabilized. Figure 7 gives the rate of super-spreader surcharge that needs to be levied on each of the banks in order that they internalize the systemic risk costs arising solely from their network centrality.

The super-spreader tax rate for each FI is obtained by multiplying the right eigenvector centrality of each node by the alpha parameter given in equation (26.a) which is then evaluated against the **#**max (´) for the system and also in terms of how much funds will accrue as shown in Table 7. To get the dollar value of the capital surcharge, the tax rates are multiplied by the Tier 1 capital of each FI. The minimum benchmark for what needs to be escrowed in a super-spreader fund will be called the super-spreader *lite* fund. The *lite* fund aims to escrow the maximum that will be lost in the first iteration (q=1) of any failed bank. Though JP Morgan can trigger the faster growing contagion with a loss of $350 billion within 3 iterations (see, Figure 5.a), the *lite* fund aims to escrow the maximum losses triggered in the first round by any FI and as shown in Table 5 this is $40 billion lost by HSBC Group.

The last row of Table 7 shows the amounts the super-spreader escrow fund can obtain by using different tax rates on the 20 FIS with the highest systemic risk indices based on their right eigenvector centrality. Remarkably, the *lite* super-spreader fund option that secures enough liquidity to buffer first tier losses which is a relatively small sum of $40bn implies tax rates of 8%-9% for the 3 FIs with the highest systemic risk index, followed by 5 FIs taxed at 5% , 3 at 2.5% and the remaining at 1% or so. This is marked in Figure 7 and highlighted in Table 7.

**Figure 7: Individual FI Tax Rates Obtained By Multiplying Right Eigenvector Centrality by or Different Values of alpha**  **> 0 (Using Equation 26.a)**

A full on stabilization package that brings the system to a **#**max <  or alternatively escrow $330 bn simply cannot be achieved. Table 7 shows that the highest level of stabilization technically feasible at **implies tax rates of over 25 % indicating the general unsustainability of the extant levels of liabilities in the derivatives markets.

**Table 7: Super-Spreader Tax Raised From Top 20 SIFIs (All columns other than Eigenvector Centrality EVC $ Billions)**Note EVC is Right Eigenvector Centrality ; Tax % = EVC x alpha; Tax$s= Tax Rate x Tier 1 Capital

Alpha

0.1

0.2

0.3

0.4

0.5

0.6

Bank Name

Tier 1 Capital

EVC

Tax

Tax

$s

Tax

Tax $s

Tax

Tax

$s

Tax

Tax

$s

Tax

Tax $s

Tax

Tax $s

Barclays

77.56

0.43

4.3%

3.35

8.6%

6.69

12.9%

10.04

17.3%

13.38

21.6%

16.73

25.9%

20.08

JPMorgan

96.37

0.39

3.9%

3.77

7.8%

7.54

11.7%

11.31

15.6%

15.08

19.6%

18.85

23.5%

22.62

HSBC

35.48

0.39

3.9%

1.38

7.8%

2.75

11.6%

4.13

15.5%

5.51

19.4%

6.88

23.3%

8.26

Credit Agricole

44.53

0.26

2.6%

1.15

5.2%

2.31

7.8%

3.46

10.4%

4.62

13.0%

5.77

15.5%

6.92

BNP Paribas

90.37

0.25

2.5%

2.30

5.1%

4.61

7.6%

6.91

10.2%

9.21

12.7%

11.51

15.3%

13.82

BoA

111.92

0.25

2.5%

2.83

5.1%

5.66

7.6%

8.50

10.1%

11.33

12.7%

14.16

15.2%

16.99

Credit Suisse

39.49

0.24

2.4%

0.96

4.9%

1.93

7.3%

2.89

9.8%

3.86

12.2%

4.82

14.7%

5.79

Deutsche Bank

49.42

0.21

2.4%

1.02

4.9%

2.05

7.3%

3.07

9.8%

4.10

12.2%

5.12

14.7%

6.15

UBS

42.32

0.19

1.9%

0.82

3.9%

1.64

5.8%

2.47

7.8%

3.29

9.7%

4.11

11.7%

4.93

Uni Credit

56.07

0.19

1.9%

1.08

3.8%

2.16

5.8%

3.23

7.7%

4.31

9.6%

5.39

11.5%

6.47

Citibank

96.83

0.16

1.6%

1.56

3.2%

3.13

4.8%

4.69

6.5%

6.25

8.1%

7.81

9.7%

9.38

Societe General

34.69

0.16

1.6%

0.55

3.1%

1.09

4.7%

1.64

6.3%

2.18

7.9%

2.73

9.4%

3.28

Goldman Sachs

17.15

0.15

1.5%

0.27

3.1%

0.53

4.6%

0.80

6.2%

1.06

7.7%

1.33

9.3%

1.59

Wachovia

39.79

0.13

1.3%

0.53

2.7%

1.06

4.0%

1.58

5.3%

2.11

6.6%

2.64

8.0%

3.17

Lloyds

74.27

0.13

1.3%

0.98

2.6%

1.96

3.9%

2.93

5.3%

3.91

6.6%

4.89

7.9%

5.87

Morgan Stanley

46.67

0.12

1.3%

0.54

2.6%

1.08

3.9%

1.61

5.3%

2.15

6.6%

2.69

7.9%

3.23

Dexia

25.24

0.06

0.6%

0.15

1.2%

0.31

1.8%

0.46

2.5%

0.62

3.1%

0.77

3.7%

0.93

Standard Chartered

24.58

0.05

0.5%

0.13

1.0%

0.25

1.5%

0.38

2.0%

0.50

2.6%

0.63

3.1%

0.75

Wells Fargo

43.77

0.05

0.5%

0.21

1.0%

0.42

1.4%

0.63

1.9%

0.85

2.4%

1.06

2.9%

1.27

NY Mellon

10.15

0.05

0.5%

0.05

1.0%

0.10

1.4%

0.15

1.9%

0.20

2.4%

0.24

2.9%

0.29

RBS

98.28

0.04

0.4%

0.41

0.8%

0.83

1.3%

1.24

1.7%

1.65

2.1%

2.07

2.5%

2.48

Superspreader fund

24.04

48.09

72.13

96.17

120.21

144.3

**5. Conclusion**

A major hallmark of SIFIs is their activity in financial derivatives markets. The analysis of this paper has one clear message. The global derivatives markets in the post Lehman period, despite considerable compression of bilateral positions, are unstable and it can bring about catastrophic failure. Quite simply a threat of failure to any of the SIFIs is an immediate threat to the others, solely from their derivatives assets and liabilities. The network topology where the very high percentage of exposures is concentrated among a few highly interconnected banks implies that they will stand and fall together. This topological fragility of the derivatives markets as risk sharing institutions has an implicit moral hazard problem that undermines their social usefulness. To date, apart from Craig and von Peter (2010) and Fricke and Lux (2012), insufficient attention has been paid by financial network modelers to the highly sparse financial networks especially prominent in the derivatives markets and also on the manner in which contagion propagates in the central clustered core of the network. The empirically calibrated network for derivatives liabilities manifests a highly clustered core-periphery structure andextreme form of *too interconnected to fail* (TITF). This characteristic form of contagion propagation has been quantified in Figures 5.a- 5.f in the case of initial failure of a number of different banks that belong to the core and those that have out-degrees to the core. The implied socialized losses are very large (to the tune of $350 billion) and the liabilities arising from extant derivatives network structures cannot be supported by the existing capital base. The good news is that the highly clustered network structure permits targeted management of systemic risk. One of the main contributions of the paper is to use network analysis to design a set of surcharges that will enable the SIFIs in the derivatives markets to internalize the costs imposed on other FIs and also the tax payer by their failure.

Studies that have used network analysis for the analysis and management of systemic risk of financial systems have investigated a plethora of network statistics. However, unless these statistics relate to how instability becomes endemic in the financial network, it is hard to devise a metric of instability for the system as a whole and show how each of the FIs contributes to system instability. This paper is the first to show how network stability which is based on the maximum eigenvalue of a non-positive financial matrix of liabilities relates to the right eigenvector centrality of FIs and their contagion spreading capabilities in the system. Unlike, most network based epidemic models, typically the rate of ‘infection’ between two nodes in the financial system is pair-wise heterogeneous. The minimum ‘cure’ rate often specified by regulators (the threshold  in the model), on the other hand, is the same for all FIs. Clearly, this has not been designed to help FIs to internalize the costs of their failure on others. The right eigenvector centrality has been identified as providing the systemic risk index for each FI while the left eigenvector centrality yields the exposure or vulnerability index of FI. The right eigenvector centrality is not merely a measure based on the number of weighted out degrees a FI has, but also the extent to which it is connected to other highly central nodes. Indeed, it was seen that even FIs with few out degrees but those that had liabilities to the highest scoring ‘rich club’ members will also have a relatively high systemic risk index and can threaten the system with contagion in exactly the same way as those in the core, albeit over a longer time frame.

A progressive capital surcharge rate based on the right eigenvector centrality of each FI in the pre-tax setting is designed and calibrated in Section 4.4 in order to achieve a much more stable system with a reduced maximum eigenvalue.[[33]](#footnote-34) As in the case of the 2009 Q4 derivatives liabilities data, a full on stabilization fund was both technically infeasible and economically exorbitant in cost. The hierarchical core-periphery structure of networks of liabilities in off balance sheet financial interactions, enables the design of a super-spreader *lite* escrow account based on the surcharges on 20 FIs with non-negligible right eigen-vector centrality. The objective of the *lite* fund is to be able to cover the maximum direct first round losses that could be inflicted by a failed FI which is estimated to about $40bn.

Operationalizing macro-prudential policy is now well understood to revolve around the following. First, how to measure systemic risk of a financial system? While a whole array of systemic risk factors can be listed (see, Markose, 2011) which include perverse incentives provided by poorly designed regulation, many accept that first step is to quantify systemic risk within the financial sector from the activities of financial firms themselves. The overview given in Section 1 of the market data based single index metrics of systemic risk indicates that this is a flawed enterprise in that such a systemic risk index is prone to the paradox of stability. This was first pointed by Hyman Minski (1982) and is now being rediscovered by Borio and Drehman (2009) and others. Extrapolating from individual VaR estimates to Co-Var as a measure of systemic risk and then attempting to counteract procyclicality of market based data especially on default probabilities which are at their lowest when the financial boom is peaking, is a fraught exercise.[[34]](#footnote-35) This paper recommends that direct bilateral balance sheet and off balance data of FIs is visualized using network models with the least amount of tainting by market price based metrics. As noted in Section 3.2, the growth of liabilities to Tier 1 capital ratio of a FI which will increase the maximum row sum of such a matrix will increase the instability of the system in terms of its maximum eigenvalue. At the time when a FI is increasing its market share by expanding its liabilities, it could succeed in increasing short term profits and unless there is a holistic visualization of what is happening to the system wide eigen-pair, alarm bells are unlikely to go off. From a pragmatic point of view such digital financial network mapping is already being developed for regulatory purposes at the Reserve Bank of India and also at the Mexican Central bank. Hernando de Soto (2012) has made a case that a public registry of ownership of intangibles such as financial assets is the final frontier of private property owning capitalistic system if the system is not to revert to the twilight zone of unaccounted black markets.

The second aspect of operationalizing systemic risk is on how to measure the contributions of individual FIs to the systemic risk. As discussed at length in Section 3.2, the eigen-pair of a non-negative real matrix of net liabilities to capital ratio of FIs provides a natural framework to relate an FI’s contribution to systemic risk. There is a straightforward logic of using only the matrix of liabilities to Tier 1capital ratio data to levy surcharges based on a FI’s eigen-vector centrality in it. The procedure is fully transparent and it avoids holding the FI culpable for contagion from second order fire sales and other market mark downs.

Thirdly, how to design prudential requirements on FIs, such as capital surcharge, tax or fee for a financial stability fund, based on their systemic risk contributions? What the analysis of systemic risk based on the maximum eigen-pair of the bilateral (netted) liabilities matrix demonstrates is that any homogeneous threshold rate will not enable SIFIs to internalize the systemic risk costs. Also, it is unlikely that SIFIs can internalize systemic risk costs arising from their activities in specific markets with the aid of generic surcharges. In the context of the post 2007 discourse, it will be a mistake to simply focus macro-prudential scrutiny on so called SIFIs irrespective of the financial markets they operate in. Instead, it is important to understand how financial systems such as the global derivatives markets entail a network structure of contractual obligations and exposures that is unstable. As the escrow fund works as a performance guarantee for FIs’ derivatives liabilities, it is on par with collateral or paid up capital to a CCP. Finally, the escrow fund designed on the basis of an FI’s derivatives liabilities aggregated and netted across all derivatives products, affords the FI the best benefits of low cost clearing which may not be available in current arrangements and likely developments of multiple centralized clearing platforms (see, Singh (2011)). This may provide incentives for participants of OTC markets to submit a full set of data on derivatives payables and receivables vis-á-vis all counterparties and products to the relevant regulatory authority.

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Appendix : Table A.1  
**Financial Derivatives for the top 22 banks ($ billion)**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Gross Notional | GPFV | GNFV | Derivative Assets | Derivative Liabilities | Total Assets | Tier 1 Capital |
| JP Morgan Chase | 78,665 (11.66%) | 1,550  (14.81%) | 1,517  (14.95%) | 95  (8.16%) | 81  (9.15%) | 2,692  (7.23%) | 96  (6.82%) |
| Royal Bank of Scotland | 76,377  (11.33%) | 690  (6.59%) | 663  (6.53%) | 106  (9.09%) | 81  (9.08%) | 2,379  (6.39%) | 98  (6.96%) |
| Bank of America | 72,529  (10.76%) | 1,494  (14.28%) | 1,462  (14.41%) | 81  (6.91%) | 44  (4.91%) | 2,855  (7.66%) | 112  (7.92%) |
| Deutsche Bank | 71,040  (10.53%) | 867  (8.28%) | 834  (8.22%) | 100  (8.53%) | 67  (7.58%) | 2,156  (5.79%) | 49  (3.50%) |
| Barclays Group PLC | 61,144  (9.07%) | 649  (6.21%) | 628  (6.19%) | 65  (5.57%) | 49  (5.49%) | 2,155  (5.78%) | 78  (5.49%) |
| BNP Paribas | 59,697  (8.85%) | 522  (4.99%) | 493  (4.86%) | 60  (5.17%) | 31  (3.54%) | 2,956  (7.93%) | 90  (6.40%) |
| Credit  Suisse | 44,550  (6.61%) | 735  (7.02%) | 734  (7.23%) | 59  (5.07%) | 60  (6.79%) | 1,066  (2.86%) | 39  (2.80%) |
| Morgan Stanley | 41,506  (6.15%) | 928  (8.86%) | 890  (8.77%) | 49  (4.20%) | 38  (4.30%) | 773  (2.08%) | 47  (3.30%) |
| Goldman Sachs | 41,118  (6.10%) | 654  (6.25%) | 591  (5.83%) | 45  (3.88%) | 24  (2.71%) | 182  (0.49%) | 17  (1.21%) |
| Citibank | 36,087  (5.35%) | 706  (6.74%) | 690  (6.80%) | 87  (7.46%) | 53  (5.95%) | 1,764  (4.74%) | 97  (6.85%) |
| UBS Bank Global | 28,013  (4.15%) | 436  (4.16%) | 424  (4.18%) | 74  (6.31%) | 66  (7.45%) | 1,385  (3.72%) | 42  (3.00%) |
| Credit Agricole | 21,698  (3.22%) | 351  (3.36%) | 348  (3.43%) | 70  (5.99%) | 57  (6.42%) | 2,237  (6.01%) | 45  (3.15%) |
| HSBC Group | 14,477  (2.15%) | 247  (2.36%) | 248  (2.44%) | 58  (4.95%) | 58  (6.53%) | 1,175  (3.15%) | 35  (2.51%) |
| Unicredit | 4,978  (0.74%) | 110  (1.05%) | 112  (1.10%) | 89  (7.58%) | 90  (10.11%) | 1,334  (3.58%) | 56  (3.97%) |
| Lloyds | 3,784  (0.56%) | 60  (0.58%) | 50  (0.49%) | 29  (2.51%) | 16  (1.77%) | 1,605  (4.31%) | 74  (5.26%) |
| Societe General | 3,601  (0.53%) | 254  (2.43%) | 251  (2.48%) | 26  (2.10%) | 12  (1.97%) | 1,150  (3.09%) | 35  (2.46%) |
| ICE Trust U.S. | 3,302  (0.49%) | 4  (0.04%) | 4  (0.04%) | 0  (0.00%) | 0  (0.00%) | 6  (0.02%) | 0  (0.00%) |
| Standard Chartered | 2,905  (0.43%) | 38  (0.36%) | 37  (0.36%) | 8  (0.66%) | 6  (0.68%) | 437  (1.17%) | 25  (1.74%) |
| Dexia | 2,629  (0.39%) | 44  (0.42%) | 46  (0.45%) | 12  (1.00%) | 14  (1.52%) | 830  (2.23%) | 25  (1.79%) |
| Wachovia | 2,263  (0.34%) | 66  (0.63%) | 66  (0.65%) | 29  (2.46%) | 9  (1.04%) | 997  (2.68%) | 40  (2.82%) |
| Bank of New York Mellon | 1,275  (0.19%) | 16  (0.16%) | 17  (0.16%) | 8  (0.71%) | 9  (1.06%) | 250  (0.67%) | 10  (0.72%) |
| Wells Fargo | 1,073  (0.16%) | 16  (0.16%) | 15  (0.15%) | 14  (1.20%) | 9  (0.98%) | 1,217  (3.27%) | 44  (3.10%) |
| Others | 1,659  (0.25%) | 27  (0.26%) | 26  (0.26%) | 30  (2.59%) | 26  (2.94%) | 5,650  (15.17%) | 258  (18.24%) |

1. Sheri Markose ([scher@essex.ac.uk](mailto:scher@essex.ac.uk)) is a professor in the Economics Department of the University of Essex, UK. This project was undertaken for the IMF Monetary and Capital Markets Department from 28 March -30 December 2011. This draft of the paper reflects feedback received from participants at the 7 December 2011 presentation given at the IMF and also at the Financial Stability Group of the Bank of England on 13 February 2012. This includes discussions with Aditya Narain, David Bholat, Jorge Chan- Lau, Karl Habermeier, Laura Kodris, Jodi Scarletta, Anna Ilyina, Andre Santos and Brenda Gonzalez. I’m grateful for initial discussions with Inci Otker Robe and Manmohan Singh which helped set the objectives of the project. I thank Daniel Fricke and Sitabhra Sinha for discussions on network structures. Research assistance from Edna Solomon was vital in the collection of data. The software for computational and visual outputs in this project was developed by me jointly with Simone Giansante and Ali Rais Shaghaghi. All errors and omissions are the author’s sole responsibility. [↑](#footnote-ref-2)
2. Of the $170 billion US tax payer bailout of the AIG, the initial $85 billion payment to AIG was geared toward honouring its CDS obligations totalling over $66.2 billion. These include payouts to Goldman Sachs ($12.9 billion), Merrill Lynch ($6.8 billion), Bank of America ($5.2 billion), Citigroup ($2.3 billion) and Wachovia ($1.5 billion). Foreign banks were also beneficiaries, including Société Générale and Deutsche Bank, which each received nearly $12 billion; Barclays ($8.5 billion), and UBS ($5 billion). (Source: 15 March 2009 press release “AIG Discloses Counterparties to CDS, GIA and Securities Lending Transactions”). Likewise, the Standard and Poor Report of August 2008, attributes the take-over of Merrill Lynch by Bank of America in 2008 to the failure of a little known Monoline called ACA to deliver CDS cover of $5 billion on Merrill’s balance sheet mortgage backed securities which had a total of $18.8 billion of CDS guarantees. [↑](#footnote-ref-3)
3. The gross notional value of the CDS obligations of Lehman Brothers, ranked the 10th largest counterparty, is placed at between $5trillion and $3.65trillion (Financial Times, 15 Sept 2008). The novation losses of CDS contracts for which Lehman was a counterparty are given at between £30 billion- $50 billion. Of the $400 billion CDS with Lehman Brothers itself as the reference entity on a face value of its bonds of $158 billion had recovery rate of as little as 8.625 cents per dollar, the net CDS settlement on this was about $6 billion. Given the very small percentage of CDS positions being hedges on Lehman bonds, Das (2010), the direct losses on Lehman bonds were close to $150 billion. [↑](#footnote-ref-4)
4. BIS Quarterly Review Statistical Annex, June 2010, Table 19. [↑](#footnote-ref-5)
5. The Q4 2009 OCC Quarterly Report, in Table 3 also gives the split between exchange traded and OTC derivatives activity for US FDIC commercial banks and Trust Companies, respectively, as 3.9% ($8.23 trillion) and 96.1% ($204.58 trillion). However, when the estimates are done for the top 25 US Bank Holding Companies (Table 2 of Consolidated Financial Statements for Bank Holding Companies, FRY-9 Schedule HC-L) there is considerable increase in derivatives activities for Morgan Stanley ($41.505 trillion with $1.822 trillion in exchange traded), Bank of America ($28.213trillion with $2.26 trillion of this exchange traded) and non FDIC entities of Taunas Corporation, Barclays Group US, GMAC and MetLife (totalling $1.91 trillion of which $478.348 million is exchange traded). [↑](#footnote-ref-6)
6. Table 23A BIS Quarterly Review Statistical Annex, June 2010. Centrally cleared credit derivatives (not to be confused with exchange traded derivatives which furnish pre trade price transparency) which include indices, sovereigns and single name corporate reference entities are mostly being cleared by ICE Clear Credit in the US and ICE Clear in Europe. Since their respective launches in March 2009 and July 2009, to date a total of $23.73 trillion gross notional CDS has been centrally cleared. For June end 2010, the BIS Quarterly Review (December 2010) shows that the CCP is the counterparty to $3.3 trillion gross notional CDS contracts. [↑](#footnote-ref-7)
7. <http://www.scribd.com/doc/37557210/Fitch-Market-Research-Global-Credit-Derivatives-Survey-09162010> . [↑](#footnote-ref-8)
8. These are Goldman Sachs, JP Morgan Chase, Barclays, Bank of America, Deutsche Bank, Morgan Stanley, Credit Suisse, BNP Paribas, UBS, Bank of America, Merrill Lynch, Royal Bank of Scotland. [↑](#footnote-ref-9)
9. See <http://www.financialstabilityboard.org/publications/r_111104bb.pdf> [↑](#footnote-ref-10)
10. Reflexivity refers to fixed point mappings which include wrong way risk (CDS protections providers themselves get downgraded as conditions on the credit risk on underlying deteriorates) and can exacerbate tail risk that derivatives are meant to mitigate. Fitch Ratings’ Global Credit Derivatives Surveys and 2009 DTCC data on outstanding contracts on reference entities show that seven broker-dealers in derivatives markets have moved up the ranks in the top 25 reference entities in CDS markets with many of them providing protection on others. Duplicating contracts in a bilaterally offsetting manner with a Centralized Clearing Platform do not erase the original interconnections between counterparties. [↑](#footnote-ref-11)
11. Graph 5B of ***OCC Quarterly Report on Bank Trading and Derivatives*** Q4 2009 shows that there has been a greater growth of *netting benefits* relative to the growth in gross notional value of derivatives contracts. See also Das (2010). [↑](#footnote-ref-12)
12. See Kirman *et al* (2007) for a discussion of the dimensionality problem that this entails in the use of Myerson-Shapley principle when instead of sub-groups we have sub-networks. [↑](#footnote-ref-13)
13. As credit growth boosts asset prices, CDS spreads and VIX indices which are inversely related to asset prices are at their lowest precisely before the crash when asset prices peak. At the 2010 IMF Workshop on *Operationalizing Systemic Risk*, Rama Cont and Sheri Markose strongly objected to the use of a statistical index based on market price information (on CDS spreads) to provide an early warning signal for banking fragility. Markose(2010) showed that the Sergoviano-Goodhart (2009) Banking Stability Index spiked contemporaneously with the crisis, same as any other market index such as the VIX or VFTSE and it also subsided far too soon after the Lehman Brother crisis making it devoid of requisite information as an early warning signal for impending and prolonged financial market crisis. [↑](#footnote-ref-14)
14. To reverse the procyclical consequence of contemporaneous market prices in a market price based systemic risk measure, Adrian and Brunnermeir (2009) in the forward Co-VAR include panel data on lagged FI specific characteristics such as size, leverage, maturity mismatch etc. This supplemented quantile regressions for forward Co-Var produced was found to be inversely related to the systemic risk measure based on market price data alone.

    [↑](#footnote-ref-15)
15. See, Reserve Bank of India Financial Stability Report, December 2011. [↑](#footnote-ref-16)
16. This is named after the work of Stanley Milgram (1967) on the six degrees of separation in social networks. It has been found that globally on average everybody is linked to everybody else by no more than six indirect links. [↑](#footnote-ref-17)
17. Numerically, Ei , the connected edges is calculated as follows. Using the NxN adjacency matrix X = (aij)N with aij=1 (aij1 , for short) if there is a link between *i* and *j* and aij=0, if not. Agent *i*’s *ki* neighbours Ξi  = { ∀ j , j ≠ i, aij = 1} , Ei  for a directed graph is calculated as Ei =  , j ≠ m. [↑](#footnote-ref-18)
18. In the B-A preferential attachment algorithm, a node i receives a new link with probability pi = . In contrast, the Watts and Strogatz algorithm for the construction of small world networks, starts with a regular graph and then has random reconnections for a subset of outdegrees of each node. This obtains the short path length and retains high clustering of the regular graph, but fails to obtain the scale free degree distribution with a few very highly connected nodes. Hence, following Zhou and Mondragon (2003), it is the B-A algorithm that is modified to achieve the core-periphery network with small world properties. [↑](#footnote-ref-19)
19. The condition of the Perron-Frobenius theorem that guarantees a positive eigenvector corresponding to the maximum eigenvalue for the non-negative matrix  is that the directed graph it represents should be irreducible. That is, for any randomly selected pairs of nodes (i,j) there is a path between them, viz.  is strongly connected, Meyer (2000). [↑](#footnote-ref-20)
20. Fujiwara *et. al.*(2009) have also noted the significance of the right and left eigenvectors to provide centrality indices for banks and firms in a birpartite graph of lending relationships between banks and firms in Japan. [↑](#footnote-ref-21)
21. In a complete graph, if bank i’s total exposure is equally divided among its N-1 counterparties, then risk is shared equally at the rate of 1/N-1. The demise of a single counterparty has a very small impact on i. In contrast, in Allen and Gale’s incomplete circle network where each bank is exposed to only one other for the full 100% of its receivables, then the failure of any bank in the circle will bring the others down. A measure of perfect homogeneity of a network is one where row sums are equal. [↑](#footnote-ref-22)
22. For a recent criticism of the entropy method in the construction of networks, see, the 2010 ECB Report on *Recent Advances in Modeling Systemic Risk Using Network Analysis*. [↑](#footnote-ref-23)
23. Karl Habermeier raised this exact point in a recent discussion with the author. [↑](#footnote-ref-24)
24. Note, as shown in Kyriakopoulos *et. al.* (2010) the network mapping of electronic real time payment and settlement systems is highly sensitive to the time scale over which flows are estimated. This problem is not something that has been resolved yet. [↑](#footnote-ref-25)
25. Node strength is given by the row sum of a FI’s activities in matrix  and as will be seen in the next section, those FIs with large and highly divergent row sums from the mean can exert destabilizing impact on the system. [↑](#footnote-ref-26)
26. In the epidemiology literature (see, Wang *et. al.* (2003))  is the cure rate and (1-) is the rate of not surviving. [↑](#footnote-ref-27)
27. Note, from equation (10) Ciq+1 = Ciq +  (Ci0 – Ciq) - , that is bank i’s Tier capital at q+1 is given by its capital at q, plus remaining reserves less the losses from failed counterparties at q. If this is zero or negative, clearly the bank i fails at q+1. This can be derived from (10) by noting that (1-Ciq/Ci0) = 1- - Ciq/Ci0 +  Ciq/Ci0. [↑](#footnote-ref-28)
28. Note if **e** is a nx1 column vector the 1xn the row vector is **e**´= [e1, e2, e3, … en]. Then, **e e**´= **E** is a nxn matrix and  **e**´ **e =** is the scalar product. [↑](#footnote-ref-29)
29. In Wang *et. al.* (2003) , where there is a homogenous infection rate β and cure rate ρ, the stability condition is that max < ρ/β or alternatively that β/ρ < 1/max. The latter is called the epidemic threshold and if the condition is violated, contagion becomes endemic. Note, here max is of th adjacency matrix. [↑](#footnote-ref-30)
30. For obvious reasons the Giakkoupis *et. al.* (2005) algorithm that reduces contagion in network by reducing the maximum eigenvalue is called the EIG algorithm. [↑](#footnote-ref-31)
31. Here, denote the eigenvector obtained by the norm ||.||∞ as **v1||.||∞**  and the one by the norm ||.||2 as **v1**. Let g be a constant and g **v1||.||∞ =**  **v1.** Then g2 **v1||.||∞ ′** **v1||.||∞** = **v1′ v1 =** 1. This gives g = (1/ **v1||.||∞ ′** **v1||.||∞** )1/2. Note, **v1||.||∞ ′** **v1||.||∞** is simply the sum of squares of the elements of the vector **v1||.||∞ .** [↑](#footnote-ref-32)
32. Clearly, it is possible to apply the capital surcharge in denominator of (23),  
     *Si#* = . This was, in fact, tried out in an earlier draft of the paper. But following the principle of ‘greedy’ algorithms as in the EIG algorithm of Giakkoupis *et. al.* (2005) that seek the largest possible reductions of the maximum eigenvalue of the system, the current proposal in (24) was found to be more effective. Note, the algorithm applied in equation (24) was constrained to retain a non-negative matrix for ´#(). [↑](#footnote-ref-33)
33. As noted (see footnote 30) this methodology is similar to the EIG algorithm of Giakkoupis *et. al.* (2005) who found that the use of eigenvector centrality to target and inoculate super-spreaders works better than other proxies such as number of degrees of nodes. [↑](#footnote-ref-34)
34. Markose and Alentorn (2011) have suggested the use of higher moments such as negative skewness within a generalized extreme value (GEV) distribution framework to track the probability of the largest price/market drops. A traded option implied negative skew index peaks with the market and does not require an inversion of logic needed with, for example, a volatility index. [↑](#footnote-ref-35)