‘Too Interconnected To Fail’ Financial Network of US CDS Market: Topological Fragility and Systemic Risk

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Abstract

A small segment of credit default swaps (CDS) on residential mortgage backed securities (RMBS) stand implicated in the 2007 financial crisis. The dominance of a few big players in the chains of insurance and reinsurance for CDS credit risk mitigation for banks’ assets has led to the idea of *too interconnected to fail (TITF)* resulting, as in the case of AIG, of a tax payer bailout. We provide an empirical reconstruction of the US CDS network based on the FDIC Call Reports for off balance sheet bank data for the 4^{th} quarter in 2007 and 2008. The propagation of financial contagion in networks with dense clustering which reflects high concentration or localization of exposures between few participants will be identified as one that is *TITF*. Those that dominate in terms of network centrality and connectivity are called ‘super-spreaders’. Management of systemic risk from bank failure in uncorrelated random networks is different to those with clustering. As systemic risk of highly connected financial firms in the CDS (or any other)

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financial markets is not priced into their holding of capital and collateral, we design a super-spreader tax based on eigenvector centrality of the banks which can mitigate potential socialized losses.

Keywords: Credit Default Swaps, Financial Networks, Eigenvector Centrality, Financial contagion, Systemic Risk, Super-spreader tax

1. Introduction

The 2007 financial crisis which started as the US 'sub-prime' crisis, through a process of financial contagion led to the demise of major banks and also precipitated severe economic contraction the world over. Since 2008, tax payer bailout and socialization of losses in the financial system has transformed the banking crisis into a sovereign debt crisis in the Euro zone. In the 2002-2007 period, credit risk transfer (CRT) from bank balance sheets and the use of credit derivatives to insure against default risk of reference assets has involved big US banks and non-bank FIs in the credit derivatives market which is dominated by credit default swaps (CDS). This market has become a source of market expectations on the probability of default of the reference entity which since 2008 has increasingly included high CDS spreads on sovereigns and FIs. Banks are major protection buyers and sellers in this market and have become vulnerable as a result. Due to inherent structural weaknesses of the CDS market and also those factors arising from poor regulatory design, as will be explained, CDS which constitute up to 98% of credit derivatives have had a unique, endemic and pernicious role to play in the 2007 financial crisis. This paper will be concerned with modelling a specific weakness of CDS which is also well known for other modern risk sharing institutions involving over-the-counter (OTC) financial derivatives, and this pertains to the heavy concentration of derivatives activities among a few main participants.

The key elements of financial crises, the case of 2007 financial crisis being no exception, is the growth of innovations in private sector liquidity and leverage creation which are almost always collateralized by assets that are procyclically sensitive, viz. those that lose value with market downturns.[2]

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[2] The use of procyclical RMBS assets as collateral for bank liabilities in asset backed commercial paper (ABCP) conduits in the repo market is given as a fundamental reason for the contraction of liquidity and the run on the repo markets in the 2007 crisis, Gorton

The specific institutional propagators of the 2007 crisis involved residential mortgage backed securities (RMBS) which suffered substantial mark downs with the collapse of US house prices.[3] Then it was a case of risk sharing arrangements that went badly wrong. This came about due to the role of CDS in the CRT scheme of Basel II and its precursor in the US, the Joint Agencies Rule 66 Federal Regulations 56914 and 59622 which became effective on January 1, 2002. This occurred in the context of synthetic securitization and of Collateralized Mortgage Obligations (CMO) which led to unsustainable trends and to systemic risk. Both holders of the RMBS and CMO assets in the banking sector and those servicing credit risk via the CDS market (cf. American Insurance Group (AIG)) required tax-payer bailouts.[4]

The Basel II risk weighting scheme for CRT of assets on bank balance sheets and its forerunner in the US which set out the capital treatment in the Synthetic Collateralized Loan Obligations guidance published by the Office of Comptroller of the Currency (OCC 99-43) for the 2002 Joint Agencies Rule 66, stand implicated for turbo charging a process of leverage that increased connectivity between depository institutions and as yet unregulated non-depository financial intermediaries and derivatives markets. Under Basel I since 1988, a standard 8% regulatory capital requirement applied to banks with very few exceptions for the economic default risk of assets being held by banks. In the run up to Basel II since 2004 and under the 2002 US Joint Agencies Rule 66, the 50% risk weight which implied a capital charge of 4%

(2009). The loss of confidence arising from the uncertainty as to which bank is holding impaired RMBS assets that were non-traded, typically called a problem of asymmetric information, exacerbated the problem.

[3] See, Brunnermeier (2009), Stulz (2010), Ashcroft and Schuermann (2008) and Gorton and Metrich (2009). They, respectively, cover the unfolding phases of the crisis, the specific characteristics of credit derivatives, the features relevant to sub-prime securitization and the collateralized debt obligations.

[4] Kiff et al. (2009) place the size of increased collateral calls on AIG's CDS guarantees following its ratings downgrades at a relatively modest $15 bn that is was unable to meet. While the current cost to the US tax payer of the AIG bailout stands at $170 bn, the initial $85 bn payment to AIG was geared toward honouring its CDS obligations to counterparties totalling over $66.2 bn. These include payouts to Goldman Sachs ($12.9 billion), Merrill Lynch ($6.8 bn), Bank of America ($5.2 bn), Citigroup ($2.3 bn) and Wachovia ($1.5 bn). Foreign banks were also beneficiaries, including Société Générale and Deutsche Bank, which each received nearly $12 bn; Barclays ($8.5 bn); and UBS ($5 bn). The following 15 March 2009 press release "AIG Discloses Counterparties to CDS, GIA and Securities Lending Transactions" provides useful information.

on residential mortgages could be reduced to a mere 1.6% through the process of synthetic securitization and external ratings which implied 5 times more leverage in the system.[5] In synthetic securitization and CRT, an originating bank uses CDS or guarantees to transfer the credit risk, in whole or in part, of one or more underlying exposures to third-party protection providers. Thus, in synthetic securitization, the underlying exposures remain on the balance sheet of the originating bank, but the credit exposure of the originating bank is transferred to the protection provider or covered by collateral pledged by the protection provider. This strongly incentivized the use of CDS by banks which began to hold more MBS on their balance sheets and also brought AAA players such as AIG, hedge funds and erstwhile municipal bond insurers called Monolines into the CDS market as protection sellers.[6] Only banks were subject to capital regulation while about 49% (see, British Bankers Association for 2006 for the breakdown of institutions involved as CDS protection sellers and buyers) of those institutions which were CDS sellers in the form of thinly capitalized hedge funds and Monolines,[7] were outside the regulatory boundary. This introduced significant weakness to the CRT scheme leading to the criticism that the scheme was more akin to banks and other net beneficiaries of CDS purchasing insurance from passengers on the Titanic. Indeed, a little known Monoline called ACA which failed to deliver on the CDS protection for RMBS held by Merrill Lynch is what finally led to its absorption by Bank of America.[8] Further, as cited in the ECB CDS Report (ECB, 2009, p.57-58), in its 2007 SEC filing, AIG FP (the hedge

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[5] The risk weight of 20% applies when a bank asset has CDS protection from an AAA rated guarantor.

[6] Acharya and Richardson (2010), Blundell-Wignall and Atkinson (2008), Hellwig (2010), Markose et al. (2010, 2011) have given detailed analyses of how the regulatory framework based on risk weighting of capital and CRT resulted in perverse incentives which left the financial system overleveraged and insolvent.

[7] At the end of 2007, AMBAC, MBIA and FSA accounted for 70% of the CDS contracts provided by Monolines with the first two accounting for $625 bn and $546 bn of this. The capital base of Monolines was approximately $20 bn and their insurance guarantees are to the tune of $2.3 tn implying leverage of 115.

[8] Standard and Poor Report of August 2008 states that Merrill Lynch had CDS cover from Monolines to the tune of $18.8bn and of that ACA accounted for $5bn. ACA, 29% of which was owned by Bear Stearns, along with other Monolines suffered a ratings downgrade in early 2008 and ACA demised in 2008 defaulting on its CDS obligations. ACA had $69 bn of CDS obligations and only had $425 million worth of capital.

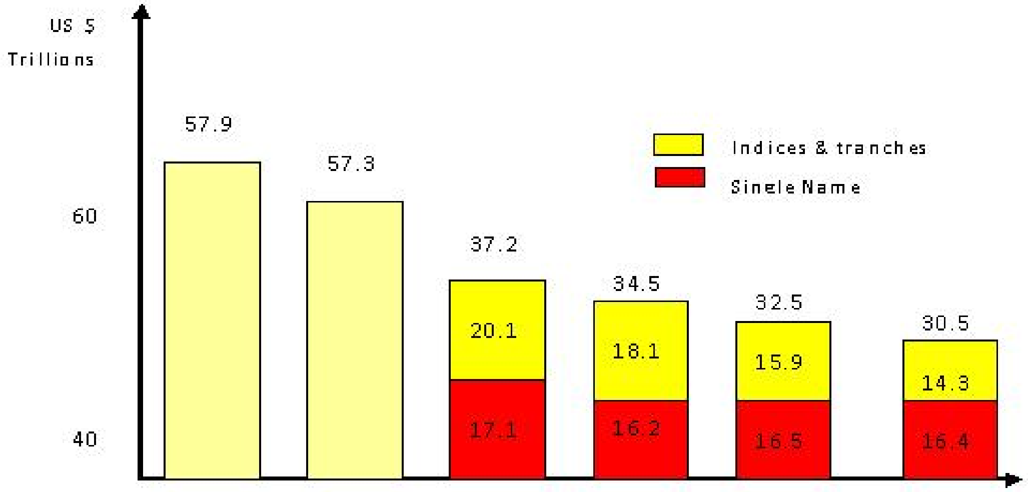

Figure 1: Credit Default Swaps Outstanding Gross Notional. Source: BIS December 07, June 08 which include all CDS contracts; DTCC for other dates record only 90% of CDS.

fund component of AIG) explicitly stated that it supplied CDS guarantees, in particular to European banks, in order for them to reduce capital requirements. The benefits that accrued to banks from CRT fell far short of the intended default risk mitigation objectives and as shown by Markose et al. (2011) participants of the CRT scheme were driven primarily by short term returns from the leveraged lending using CDS in synthetic CDOs as collateral in a carry trade.

Figure 1 shows how the CDS market peaked at about $58 trillion in the run up to the 2007 crisis. In the post Lehman period the gross notional value[9] of CDS has contracted due to the compression of CDS contracts with bilateral tear ups and a decline of CDS issuance. Tranche CDS shrank faster than single name CDS. During the short lived period of the CDO market for RMBS which peaked at over $2 trillion in 2007, about $1 trillion of the tranche based CDS was on sub-prime RMBS.

Undoubtedly, the main rationale behind CRT in the context of credit derivatives which led regulators to endorse these activities (see, e.g., IMF

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[9] Following the DTCC, the CDS notional refers to the par value of the credit protection bought or sold. Gross notional value reported on a per trade basis is the sum of the CDS contracts bought (or equivalently sold) in aggregate.

(2002); OECD (2002); IAIS (2003); BIS (2004)) is that it allows financial intermediaries (FIs) to diversify away concentrated exposures on their balance sheet by moving the risks to AAA rated institutions that seem better placed to deal with them. However, similar to the argument made by Darby (1994) about derivatives markets in general in their role in risk sharing, many have noted (see, Persuad (2002), Lucas et al. (2007), Das (2010) and Gibson (2007)) that the benefits of CRT will be compromised by the structural concentration of the CDS market. Clearly, Basel II and III schemes for CRT[10] suffer from the fallacy of composition. The premise that the transfer of credit risk from banks' balance sheets, which is a good thing from the perspective of a bank especially as the capital savings incentives allow short run asset expansion, will also lead to diversification of risk does not follow at a collective level. There is growing counterparty and systemic risk due to fragility in the network structures. Few have provided tools to quantitatively model and visualize the systemic risk consequences of what is called *too interconnected to fail (TITF)* that come with high concentration of CDS counterparties.[11] Markose et al. (2011) has pointed out that the fallacy of composition type errors can be reduced with holistic visualization of the interconnections between counterparties using financial network models. The structural signature of such financial networks given by the heavy concentration of exposures needs to be modelled and analysed to understand the network stability properties and the way in which contagion propagates in the system. In view of the growing *structural* concentration in the provision of risk guarantees through financial derivatives, we claim the topological fragility of the modern risk sharing institutions is germane to issues on systemic risk.

Given the US centric nature of the CDS market for RMBS and the fact that the FDIC Call Reports comprehensively give data on gross notional, gross positive fair value (GPFV)[12] and gross negative fair value (GNFV) of

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[10] Hellwig (2010) has correctly noted that as long as incentives for capital reduction are given for the use of CDS risk mitigants, it is business as usual in Basel III.

[11] In the publicly available slides of a study by Cont et. al. (2009), Measuring Systemic Risk in Financial Networks cited in the 2009 ECB CDS Report (ECB, 2009), Cont et. al. simulate the CDS market network connectivity and exposure sizes on the basis of the empirical properties of the Brazilian and Austrian interbank markets. We maintain that the CDS market, especially as it affects US bank solvency, has considerably more clustering and concentration risk than interbank markets.

[12] The sum total of the fair values of contracts involves the money owed to a bank by its counterparties, without taking into account netting. This represents the maximum losses

CDS for all FDIC FIs, this paper will confine the CDS network model to fit the FDIC data set. Note, the activities of the FDIC financial firms are given in their capacity as national associations rather than in terms of global consolidated holdings. The number of US FDIC financial firms involved in CDS is very few ranging from between 26-38 or so in the period since 2006 when this data has been reported. In 2006, we find that that top 5 US banks (J.P. Morgan, Bank of America, Citigroup, Morgan Stanley and Goldman Sachs) accounts for 95% of gross notional sell and over 97% in 2007 of the total CDS gross notional sell of FDIC banks. In terms of the $34 tn global gross notional value of CDS for 2008 Q4 given by BIS and DTCC, these top 5 US banks account for 92% of market share. Of the top 100 SP-500 firms surveyed by Fitch in 2009 for derivatives use[13], only 17 were found to be active in the CDS market and the top 5 US banks accounted for 96% of CDS gross notional in 2009. While the network for CDS exposures for US banks in the 2007 Q4 period showed that Monolines and insurance companies were dominant as CDS protections sellers, by 2008 Q4 we have an even greater dominance of 5 US banks in the CDS market. This came about with the demise or merger of investment banks Bear Stearns, Lehman Brothers and Merrill Lynch, contraction of CDS activities by the Monolines and the nationalization of AIG. It is a sobering fact that the origins of the financial contagion as it emanated from CDS on RMBS on US banks' balance sheets accounts for only 13% of gross notional of total US bank holdings of CDS in 2006 Q1 and falling to 7% in 2007 Q2 (see Markose et al. (2011)).

This paper is concerned with characterizing the systemic risk from this class of derivatives by considering the topology of the financial network for the counterparty exposures. Following the methods of the IBM project of MIDAS (see, Balakrishnan et al. (2010)) which aims to automate, access and visualize large financial datasets this paper will use the Markose et al. (2010) FDIC network 'visualizer' for the CDS activities of FDIC firms. One of the objectives of the paper is to highlight the hierarchical core-periphery type

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a bank could incur if all its counterparties default and there is no netting of contracts, and the bank holds no counter-party collateral. Fair values are market determined or model determined.

[13] The report by Fitch Ratings, 2009, "Derivatives: a Closer Look at What New Disclosures in the U.S. Reveal". The 100 companies reviewed were those with the highest levels of total outstanding debt in the S&P 1,500 universe. They represent approximately 75% of the total debt of S&P 1,500 companies.

structures within a highly sparse adjacency matrix to give a more precise depiction of financial firms being *TITF* in that the highly connected financial firms will bring down similarly connected financial firms implying large socialized loss of capital for the system as a whole. It aims to give a more rigorous characterization in terms of network statistics of extreme concentration of exposures between five top US banks. We will highlight the high asymmetry in network connectivity of the nodes and high clustering of the network involving a few central hub banks (sometimes called the 'rich club') which are broker-dealers in of the CDS network.

By its nature of being a negative externality, systemic risk implications of a bank's connectivity and concentration of obligations are not factored into the capital or collateral being held by banks. In a ratings based system, as succinctly pointed out by Haldane (2009), leniency of capital and collateral requirements for a few large highly rated FIs has resulted in excessive expansion of credit and derivatives activities by them which is far beyond what can be sustained in terms of system stability. Haldane (2009) calls such highly interconnected financial intermediaries 'super-spreaders' and we will retain this epithet in the financial network modelling that follows. Haldane (2009) recommends that super-spreaders should have larger buffers. We design a super-spreader tax based on eigenvector centrality of the nodes and we test it for its efficacy to reduce potential socialized losses.

Section 2 gives a brief description of CDS and discusses the potential systemic risk threats that arise from them. This includes the practice of offsetting which creates dense connections between broker-dealers. In Section 3 we will briefly review the technical aspects of network theory and the economics literature on financial networks. The main drawback of the pre 2007 economics literature on financial networks has been that models that are based on empirical bilateral data between counterparties were few in number to establish 'stylized' facts on network structures for the different classes of financial products ranging from contingent claims and derivatives, credit related interbank obligations and exposures and large value payment and settlement systems. Where bilateral data on financial exposures is not available, both empirical and theoretical models assumed network structures to be either uncorrelated random ones (see, Nier et al. (2007)) or complete network structures (see, Upper and Worms (2004)). As will be argued, these approaches crucially will not have what we call the *TITF* characteristics. While the stability of financial networks have been usually investigated using the classic Furfine (2003) algorithm, sufficient emphasis has not been given to the way in which contagion propagates in highly tiered and clustered networks and stability of the system in terms of network characteristics has not been studied. Section 4 discusses the necessary network stability results and derives the super-spreader tax fund that can mitigate potential socialized losses from the failure of highly connected banks. The super-spreader tax is based on the eigen-vector centrality of the FI in order to internalize the system wide losses of capital that will occur by its failure.

In the empirical Section 5, a quantitative analysis leading to the empirical reconstruction of the US CDS network based on the FDIC Q4 2007 and Q4 2008 data is given in order to conduct a series of stress tests that investigate the consequences of the high concentration of activity of 5 US banks. In 2007 Q4, non-bank FIs such as Monolines and hedge funds are found to be dominant in terms of eigen-vector centrality. In 2008 Q4, J.P. Morgan is identified as the main super-spreader. An equivalent uncorrelated random network equivalent in size, connectivity and total GNFV and GPFV for each bank is also constructed and systemic risk from bank failure in uncorrelated random networks is shown to be different from the empirically calibrated CDS network. Results are provided on how the super-spreader tax fund operates. Section 6 concludes the paper and outlines future work.

2. Over the Counter CDS Contracts: Potential Systemic Risk Threats

2.1. CDS Contract and Inherent Problems

A single name credit default swap is a bilateral credit derivative contract specified over a period, typically 5 years, with its payoffs linked to a credit event such as default on debt, restructuring or bankruptcy of the underlying corporate or government entity. The occurrence of such a credit event can trigger the CDS insurance payment by the protection seller who is in receipt of periodic premia from the protection buyer. Figure 2 sets out the structure of a CDS contract.

Every over the counter (OTC) CDS contract is bilaterally and privately negotiated and the respective counterparties and the contracts remain in force till the maturity date. This raises problems with regard to counterparty risk and also indicates why gross exposure matters. The periodic payments of premia are based on the CDS spread and quoted as a percentage of the gross notional value of the CDS at the start of the contract. The CDS spreads being quoted fluctuate over time. As the payoff on a CDS contract is triggered by the default on debt, the CDS spread represents, in general,

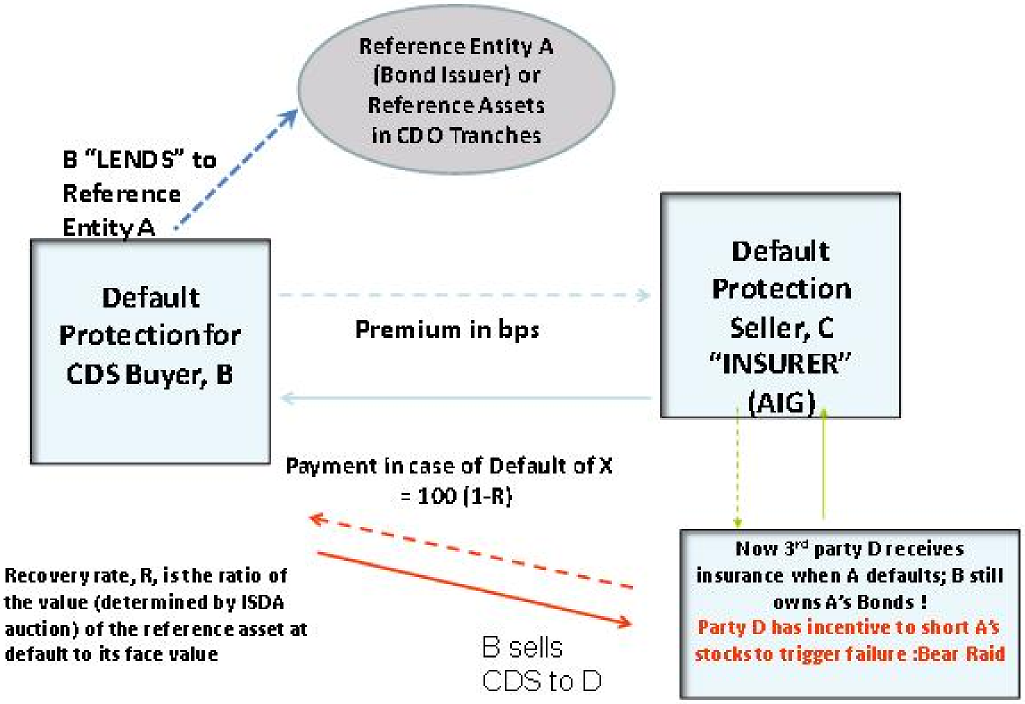

Figure 2: Credit Default Swap Structure, CDS Chain and Bear Raid. Note: Direction of CDS sale or protection guarantee is the unbroken arrow.

credit worthiness of the reference entity and specifically, the probability of default and the recovery value of the reference assets. All else being equal, higher spreads indicate growing market expectations of the default on the debt with a jump to default spike at the time of the credit event. Net CDS sellers and their counterparties holding impaired CDS reference assets may also find that CDS spreads on themselves as reference entities are adversely affected. This could hasten their own insolvency as liquidity risk in the form of the ability to raise funds is affected. This has been called 'wrong way risk'. The 2009 ECB CDS report estimated this as the correlation in the CDS spreads of CDS sellers and their respective reference entities, and finds this has grown for sellers of CDS which rely on government bailout and then sell CDS with their respective sovereigns as reference entities. Circularity of risk arises from the fact that as noted by the DTCC in December 2008, 7 top dealers are themselves among the 10 top reference entities by net protection amounts.[14]

CDS spreads are known to have strong self-reflexive properties in that they do not merely reflect the financial state of the underlying obligor, they can in turn accelerate the default event as ratings downgrade follow, cost of capital rises and stock market valuation falls for the obligor as the CDS spreads on them increase. These systemic risk factors are hard to model in formulaic CDS pricing models and hence such counterparty and circular risk are typically not modelled in CDS pricing models.

The controversial aspect about a CDS that makes the analogy with an insurance contract of limited use is that the buyer of a CDS need not own any underlying security or have any credit exposure to the reference entity that needs to be hedged. The so called naked CDS buy position is, therefore, a speculative one undertaken for pecuniary gain from either the cash settlement in the event of a default or a chance to offset the CDS purchase with a sale at an improved CDS spread. This implies that gross CDS notional values can be several (5-10) multiples of the underlying value of the debt obligations of the reference entity. It has been widely noted that naked CDS buyers with no insurable interest will gain considerably from the bankruptcy of the reference

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[14] In December 2008, the DTCC lists the following financial reference entities by net protection amounts: GE Capital ($11.074 bn), Deutsche Bank ($7.163 bn), Bank of America ($6.797 bn), Morgan Stanley ($6.318 bn), Goldman Sachs ($5.211 bn), Merill Lynch ($5.211 bn), Berkshire Hathaway ($4.632 bn), Barclays Bank ($4.358 bn), UBS($4.311 bn), RBS($4.271 bn).

entity. Note the 'bear raid' in Figure 2 refers to the possibility that when the CDS protection cover on a reference entity has been sold on to a third party, here D, who does not own the bonds of the reference entity, D has an incentive to short the stock of the reference entity to trigger its insolvency in order to collect the insurance to be paid up on the CDS. A naked CDS buy position is equivalent to shorting the reference bonds without the problems of a short squeeze that raises the recovery value of the bonds (and lowers the payoff on the CDS) when short sellers of the bonds have to 'buy back' at time of the credit event. Hence, naked CDS buying is combined with shorting stock of the reference entity. There is also the case that even those CDS buyers who have exposure to the default risk on the debt of the reference entity may find it more lucrative to cash in on the protection payment on the CDS with the bankruptcy of the reference entity rather than continue holding its debt. This is called the empty creditor phenomenon (see, Bolton and Oehmke (2011)).

Finally, as noted by Duffie et al. (2010) and as what happened in the case of the Bear Stearns hedge funds that had large CMO holdings, is that there can be a 'run' on the collateral posted by large CDS protection sellers if they suffer an actual or potential ratings downgrade. Counterparty credit risk rises to the level of systemic risk when the failure of a market participant with an extremely large derivatives portfolio could trigger large unexpected losses on its counterparties derivatives trades, which accelerates the failure of that market participant. This can be accompanied by fire sales of the collateral which can lead to significant price volatility or price distortions. Those CDS contracts operating on the ISDA (International Swaps and Derivatives Association) rules also have a provision of cross-default. If a counterparty cannot post collateral in a specified time frame, it can deem to have defaulted and if the shortfall of collateral exceeds a threshold, the counterparty is deemed to have defaulted across other ISDA CDS. These cross-defaults (a potential situation that AIG was in) can trigger a domino effect as all parties close out. Attempts at novating CDS contracts guaranteed by the 'closed out' firm especially when the underlying is potentially devalued (as in the case of RMBS assets) with other protection sellers may be difficult and if successful it increases market concentration and network fragility as now there are fewer sellers.

2.2. Broker-Dealer Concentration

The main strategy adopted by CDS dealers and counterparties to manage liquidity requirements is a practice called "offsets" which though individually rational may collectively contribute to systemic risk as the chains of CDS obligations increase and also merge. Offsets involve a strategy by which CDS participants can maximize revenue from spread trades and minimize collateral and final payouts. In Figure 2, for example, B having bought CDS cover from C, finds that the spreads have increased and may choose to eschew its hedge on the bonds of the reference entity A to earn the difference between the premia it pays to C and the higher premia it can now charge by an offset sale of CDS to D. This is marked by the red arrows in Figure 2 and is a typical spread trade. In this system, the ultimate beneficiary of CDS cover, in case of default of reference entity A, is the naked CDS buyer D. Assuming par value of \$10m for each CDS contract and zero recovery rate on reference entity bonds in Figure 2, note in the above scenario, C has an obligation to settle \$10m and then B's obligations net to zero having settled with D. We will call this an open chain or tree.

Consider the case that C offsets with D (ie. the green arrows in Figure 2 are active). We now have a closed chain of reflexive obligations (B sells to D, D sells to C and C sells to B) with the gross notional CDS value at \$30m. Should the reference entity A default, then at settlement, if *all* parties in the CDS chain remain solvent (note that B has eschewed its hedge on the reference entity), aggregate/multilateral net CDS payouts for B, C and D are zero. Zero net notional CDS value[15] gives nobody any non-premia related benefits, least of all cover on the reference entity bonds. If, however, any one of the counterparties fails, say C in a double default with the reference entity A, in the closed chain of CDS obligations, the whole chain may be brought down as B now has to face its obligation to D in terms of its gross amount

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[15] We use the DTCC definition of aggregate net notional for each reference entity, ie. the sum of net protection bought by net buyers (or net protection sold by net sellers). See, *http://www.dtcc.com/products/derivserv/data/*. This is calculated at the level of each CDS market participant and based on the gross notional of buy and sell CDS contracts, separately aggregated over all counterparties, every participant is deemed a net buyer or net seller. The net buyers (or net sellers) values are summed up to get the aggregate net notional. Note also, this assumes zero recovery rate at time of settlement. This definition of net notional involves multilateral netting while reduction of counterparty risk can arise only from what can be bilaterally netted and nullified by mutual tear ups with the failed counterparty.

of \$10 m.

Bilateral offsets and a reflexive closed chain configuration provide the most efficient *ex ante* net settlement liquidity requirements[16] *if* all counterparties deliver. Bilateral offsets on the same reference entity will reduce collateral requirements and also counterparty risk as there will be mutual tear ups when the counterparty fails. This is characteristic of network linkages in inter-dealer relationships (see, Bliss and Kaufman (2006)). It must be noted that extensive non-bilateral offsets, described above, using spread trades that aim to maximize income from CDS spreads is essential for the price discovery process. It will reduce aggregate net notional but not counterparty risk as non-bilateral offsets will result in clustered interconnections and a high level of systemic risk. Also, reduction in aggregate net notional comes at a price of reducing the aggregate capacity of the CDS market to deliver hedge benefits on reference assets.

In summary, the network topology which favours concentration of netted flows between broker-dealers is efficient in regard to liquidity and collateral requirements. It could be less stable than the one that requires more ex ante net liquidity or collateral. Liquidity or collateral provision driven from the vantage of individually rational calculations will fall short of the amounts needed for system stability (see also footnote 15). The process of offsets can nullify gross obligations if the reference entity defaults, but this requires that net CDS sellers settle. Inability to do so, can make net CDS sellers the main propagators of the financial contagion.[17] The network structure, where key CDS net sellers with large market shares have heavy CDS activity on them as reference entities, will show up as highly interconnected linkages amongst these same players. This highly interconnected multi-hub like structure that

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[16] Galbiati and Giansante (2010) have also find that networks that achieve economies in liquidity to be posted for settlement have reciprocal bilateral structures and also high interconnectivity in the form of clustering among key participants which facilitates efficient netting. Duffie and Zhu (2009) are somewhat misleading about the role of bilateral netting in the stability of the CDS market. They emphasize the savings in liquidity but, as they acknowledge, their model does not deal with so called "knock-on effects", or the problem of how the default of one CDS counterparty can lead to a chain reaction affecting others.

[17] The 2009 ECB report on CDS indicates how the potential threat from AIG was not properly identified as the Fitch survey ranked AIG as only the 20th largest in terms of gross CDS obligations and failed to note that AIG was primarily a one way seller and its sell CDS positions at \$372 bn was double the net notional amount sold by all DTCC dealers combined in October 2008.

characterizes inter-dealer CDS obligations will feature in the empirically determined CDS network model we develop.

3. Financial Network Analysis

Networks are defined by a pair of sets (N, E) which stand for nodes $N = 1, 2, 3, .., n$, and E is a set of edges. In financial networks nodes stand for financial entities such as banks, other financial intermediaries and their non-financial customers. The edges or connective links represent contractual flows of liquidity and/or obligations to make payments and receive payments. Let i and j be two members of the set N. When a direct link originates with i and ends with j, viz. an out degree for i, we say that it represents payments for which i is the guarantor. Note, an agent's out degrees corresponding to the number of its immediate neighbours is denoted by k_i. In degrees represent receivables from the bank j to the bank i. In a system of linkages modelled by undirected graphs, the relationships between N agents when viewed in $N \times N$ matrix form will produce a symmetric matrix as a link between two agents will produce the same outcome whichever of the two partners initiated it. In contrast, directed graphs are useful to study relative asymmetries and imbalances in link formation and their weights.

3.1. Bilateral Flow Matrices

3.1.1. Adjacency Matrix and Gross Flow Matrix For CDS

Key to the network topology is the bilateral relations between agents and is given by the adjacency matrix. Denote the $(N + 1) \times (N + 1)$ adjacency matrix $A = (a_{ij})^I$, here I is the indicator function with $a_{ij} = 1$ if there is a link between i and j and $a_{ij} = 0$, if not. The N^{th} agent will be represented by the US non-bank sector such as Monolines, hedge funds and insurance companies. The $N + 1^{th}$ agent represents the non-US participants. This is also used to balance the system. The adjacency matrix becomes the gross flow matrix X such that x_{ij} represents the flow of gross financial obligations from the protection seller (the row bank) to the protection buyer j (the column bank). The FDIC Call Report Data gives the Gross Negative Fair Value (GNFV) for payables and Gross Positive Fair Value (GPFV) for receivables on all CDS products that a firm is involved in with all of its counterparties. Note GNFV and GPFV is a fraction (typically by a factor of 10) of the gross notional for which the firm is a CDS seller or buyer, respectively. The total gross payables in terms of GNFV for bank i is the sum over j columns or counterparties, $G_i = \sum_j x_{ij}$ while the total gross receivables or total GPFV for each i is the sum taken across the i rows $B_i = \sum_i x_{ij}$. This is shown below :

$$
\begin{array}{ccccccc|c}
 & & & & & & & \Gamma = \sum_i G_i \\
 & 0 & x_{12} & x_{13} & ..x_{ji}. & .... & x_{1N+1} & G_1 \\
 & x_{21} & 0 & x_{23} & .... & .... & x_{2N+1} & G_2 \\
 & . & . & 0 & .... & .... & . & . \\
 & x_{i1} & . & . & 0 & & x_{iN+1} & G_i \\
\mathbf{X} = & . & . & . & & 0 & & . \\
 & x_{N+11} & . & . & x_{N+1j} & ... & 0 & G_{N+1} \\
 & & & & & & & \\
\Phi = \sum_j B_j & B_1 & . & . & B_j & .... & B_{N+1} &
\end{array}
\tag{1}
$$

The zeros along the diagonal imply that banks do not lend to themselves (see, Upper, 2007) or in this case of CDS, provide protection to themselves. There can be asymmetry of entries such that for instance G_1 need not equal B_1. However, aggregate GNFV including that of the $N+1$ entity $\Gamma = \sum_i G_i$ will be made to balance with $\Phi = \sum_j B_j$.

3.1.2. Bilaterally Netted Matrix of Payables and Receivables

Consider a matrix M with entries $(x_{ij} - x_{ji})$ gives the netted position between banks i and j. For each bank i the positive entries, $m_{ij} > 0$, in row i give the net payables vis-á-vis bank j and the sum of positive entries for bank i is its total bilaterally netted payables across all counterparties. This can be called i's CDS liabilities. The sum of the negative entries, $m_{ij} < 0$, for each bank i in the ith row gives its total bilaterally netted receivables, which is often called CDS assets.[18] Note the matrix M is skew symmetric

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[18] Note, FDIC Call Reports give the derivatives assets (liabilities) which is the GPFV (GNFV) bilaterally netted by counterparty and product and also adjusted for collateral for each bank. However, this is reported in aggregate for all derivatives products and there is no publicly available bilaterally netted data on a bank's assets and liabilities for CDS. Hence, what we will take the i^{th} bank's CDS assets and liabilities to be the sum of the

with entries $m_{ij} = -m_{ji}$. To analyse the dynamics of the cascade of failure of the ith bank on the jth one, the matrix that is relevant will only contain the positive elements of the M matrix. The direction of the contagion follows from the failed bank i owing its counterparty j more than what j owes i. Further, as we will discuss in the next section, it is customary for the net exposures of bank j to bank i relative to j's capital at time t, C_{jt}, to be greater than a threshold (signifying a proportion of j's capital) before j is said to have failed. The matrix $\mathbf{\Theta}$ that is crucial for the contagion analysis will have elements given as follows:

$$\Theta = \begin{bmatrix} 0 & \frac{(x_{12}-x_{21})^+}{C_{2t}} & \frac{(x_{13}-x_{31})^+}{C_{3t}} & .... & .... & 0 \\ 0 & 0 & \frac{(x_{23}-x_{32})^+}{C_{3t}} & .... & .... & \frac{(x_{3N}-x_{N3})^+}{C_{Nt}} \\ . & . & 0 & .... & .... & . \\ \frac{(x_{i1}-x_{1i})^+}{C_{1t}} & . & ... & 0 & ... & \frac{(x_{iN}-x_{Ni})^+}{C_{Nt}} \\ . & . & ... & ... & 0 & . \\ \frac{(x_{N1}-x_{1N})^+}{C_{1t}} & . & ... & \frac{(x_{Nj}-x_{jN})^+}{C_{jt}} & ... & 0 \end{bmatrix} \tag{2}$$

3.2. Topology of Financial Networks: Complete, Random and Uncorrelated, Correlated and Small World

Like many real world networks, namely, socio-economic, communication and information networks such as the www, financial networks are far from random and uncorrelated. In order to construct a network for the US CDS market which shows dominance of few players with a 92% and upwards of concentration of CDS exposures, we will use what are referred to as small world networks[19] (Watts (1999) and Watts and Strogatz (1998)). These networks have a top tier multi-hub of few agents who are highly connected among themselves (often called rich club dynamics) and to other nodes who

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bilaterally netted positive amounts $\sum_j (x_{ij} - x_{ji})^+$ and the sum of the bilaterally netted negative amounts $\sum_j (x_{ij} - x_{ji})^-$, respectively.

[19] This is named after the work of the sociologist Stanley Milgram (Milgram, 1967) on the six degrees of separation in social networks. It has been found that globally on average everybody is linked to everybody else in a communication type network by no more than six indirect links.

show few if any connections to others in the periphery. The properties of small world networks and how contagion propagates through them will be briefly contrasted with that for the uncorrelated Erdös-Renyi random graph and also the Barabási and Albert (1999) scale free networks.

Networks are mainly characterized by the following network statistics (a) The connectivity of a network is given by the number of connected links divided by the total number of links. There are $N(N-1)$ possible links for directed graphs and $\frac{N(N-1)}{2}$ for undirected graphs. (b) The measure of local interconnectivity between nodes is called clustering coefficient, (Δ_i denotes the clustering coefficient for node i and Δ is the coefficient for the network); (c) The shortest path length of the network estimates the average shortest path between all pairs of randomly selected nodes; and (d) Degree distribution which gives the probability distribution $P(k)$ of links of any number k, and $p(k)$ gives the probability that a randomly selected node has exactly k links. The average number of links per node is given by $< k >= \sum_k kp(k)$ and the variance of links $< k^2 >= \sum_k k^2 p(k)$. Where empirical sample data is used, $p(k) = \frac{N_k}{N-1}$ where N_k is the number of nodes with k links.

Clustering in networks measures how interconnected each agent's neighbours are and is considered to be the hallmark of social and species oriented networks. Specifically, there should be an increased probability that two of an agent's neighbours are also neighbours of one another. For each agent with k_i neighbours the total number of all possible directed links between them is given by $k_i(k_i-1)$. Let E_i denote the actual number of links between agent i's k_i neighbours, viz. those of i's k_i neighbours who are also neighbours. The clustering coefficient Δ_i for agent i is given by

$$\Delta_i = \frac{E_i}{k_i(k_i-1)} \text{ and } \Delta = \frac{\sum_{i=1}^{N} \Delta_i}{N}. \tag{3}$$

The second term which gives the clustering coefficient of the network as a whole is the average of all Δ_i's . Note that the clustering coefficient for an Erdös-Renyi random graph is $\Delta^{random} = p$ where p is the same probability for any pair of nodes to be connected. This is because in a random graph the probability of node pairs being connected by edges are by definition independent, so there is no increase in the probability for two agents to be connected if they were neighbours of another agent than if they were not. A high clustering coefficient for the network corresponds to high local interconnectedness of a number of agents in the core. In an Erdös-Renyi network, the degree distribution follows a Poisson distribution. In contrast, scale free networks have highly skewed distribution of links that follows a power law in the tails of the degree distribution, that is the probability of a node possessing k degrees is given by

$$p(k) = k^{-\alpha}, \tag{4}$$

where $\alpha > 0$ is called the power law exponent. Hence, there are some nodes which are very highly connected and many that are not. To generate power law statistics for nodes either in terms of their size or the numbers of links to/from them, Barabási and Albert (1999) proposed a process called preferential attachment, whereby nodes acquire size or numbers of links in proportion to their existing size or connectivity. This also results in a characteristic called assortativity where on average nodes will be connected with nodes with greater number of outdegrees than themselves in contrast to those with fewer.[20]

An important discovery that was made by Watts (1999) and Watts and Strogatz (1998) with regard to socio-economic networks is that while small world networks like scale free networks have in-egalitarian degree distribution with some very highly connected nodes, the central tiering of highly clustered nodes which work as hubs for the peripheral nodes (who have few direct connections to others in the periphery) is a signature feature only of small worlds. The hubs also facilitate short path lengths between two peripheral nodes. We have indicated how such a tiered structure arise in broker-dealer structures as the hub members minimize liquidity and collateral costs by implementing offsets.

Apart from the clustering coefficient, two further statistics will be used to characterize networks that show high concentration of activity and also the identification of network centrality of highly connected nodes. The first of this is the rich club coefficient. We will use the rich club coefficient, $\Phi(k)$ to identify highly connected nodes who form the club which is characterized by a fully connected network (see, Colizza et al. (2006)). The latter yields a coefficient of 1 and $k^{\#}$ will denote the critical number of out-degrees the nodes need to have to be part of the largest sized rich club with $\Phi(k) = 1$.

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[20] In the case of disassortative networks, nodes on average will be connected to those with fewer number of outdegrees than they have themselves.

The rich club coefficient is estimated as:

$$\Phi(k) = \frac{2E_{>k}}{N_{>k}(N_{>k}-1)}. \tag{5}$$

Here $N_{>k}$ refers to the number of nodes with degrees higher than a given value of k and $E_{>k}$ denotes the number of connected edges among the $N_{>k}$ nodes. The denominator divided by 2 gives the maximum number of possible edges in any direction as in an undirected graph. Monotonicity of the rich club coefficient $\Phi(k)$ in k is more likely to be true for assortative networks than disassortative ones.

The network centrality measure that has been found by the authors to correlate best with the capacity of a bank to cause the largest contagion losses on others in the Furfine (2003) type stress test is its eigenvector centrality statistic obtained for matrix $\mathbf{\Theta}$. The algorithm that determines it assigns relative centrality scores to all nodes in the network based on the principle that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes. Denoting v_i as the eigenvector centrality for the ith node, let the centrality score be proportional to the sum of the centrality scores of all nodes to which it is connected (ie. out degrees). Hence,

$$v_i = \frac{1}{\lambda}\sum_j \theta_{ij} v_j. \tag{6}$$

For the centrality measure, we take the largest real part of the dominant eigenvalue, λ_{max} , and the associated eigenvector. The i^{th} component of this eigenvector then gives the centrality score of the i^{th} node in the network. Using vector notation for this, we obtain the eigenvector equation from matrix in (2) as:

$$\mathbf{\Theta}'\mathbf{v} = \lambda_{max}\mathbf{v}. \tag{7}$$

Here $\mathbf{\Theta}'$ is the transpose of the matrix $\mathbf{\Theta}$ in equation (2). As the eigenvector of the largest eigenvalue of a non-negative matrix $\mathbf{\Theta}'$ in (2) has only positive components, positive values for the centralities are guaranteed by Perron-Frobenius theorem.

3.3. Economics Literature on Financial Networks

Pre 2007 financial network models in the economics literature have yielded mixed results. An influential and early work on connectivity in a financial network and that of financial contagion is that of Allen and Gale (2001). They gave rise to a mistaken view (see, Battiston et al. (2009)) that follows only in the case of homogenous graphs[21], ie. increasing connectivity monotonically increases system stability in the context of diversification of counterparty risk. A number of the analytical and numerically based studies in financial contagion work were confined to Erdös-Renyi random graphs such as Nier et al. (2007) and Gai and Kapadia (2010) which are interesting in terms of qualitative understanding one needs to get but as financial networks are far from random, they have some way to go.

As little empirical work has been done to date on network structures of the specific markets underpinning off-balance bank activity such as CDS responsible for triggering and propagating the 2007 crisis, it must be noted that the bulk of the empirical financial network approach has been confined to interbank markets for their role in the spread of financial contagion (see, Furfine (2003) and Upper (2011)). However, the use of the entropy method[22] (see, Upper and Worms (2004) and Boss et al. (2004)) for the construction of the matrix of bilateral obligations of banks which results in a complete network structure for the system as a whole, greatly vitiates the potential for network instability or contagion. Recent work by Craig and von Peter (2010) using bilateral interbank data from German banks have identified the tiered core-periphery structure and find that bilateral flow matrix (X) in (1) unlike in a complete or as in a Erdös-Renyi random networks is sparse in the following way:

$$X = \begin{bmatrix} \mathrm{CC} & \mathrm{CP} \\ \mathrm{PC} & \mathrm{PP} \end{bmatrix}. \tag{8}$$

Here, CC stands for the financial flows among the core banks in the centre of the network, CP stands for those between core and periphery banks, PC between periphery and core banks and PP stand for flows between periphery

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[21] In a complete graph, if bank i's total exposure is equally divided among its $N-1$ counterparties, then risk is shared equally at the rate of $\frac{1}{N-1}$. The demise of a single counterparty has a very small impact on i. In contrast, Allen and Gale (2001) consider an incomplete circle network where each bank is exposed to only one other for the full 100% of its receivables, then the failure of any bank in the circle will bring the others down.

[22] For a recent criticism of the entropy method in the construction of networks, see, the 2010 ECB Report on *Recent Advances in Modeling Systemic Risk Using Network Analysis* (ECB, 2010).

banks. The sparseness of the matrix relates to the fact that PP flows are zero and banks in the periphery of the network do not interact with one another. This structure resembles the small world network described in Section 3.1 above as being a characterization of *TITF* structure in the core of the network. Hence, the criticism Craig and von Peter level at extant financial networks literature is worth stating here. They say that many interbank models proposed in the economics literature (e.g. Allen and Gale (2001), Freixas et al. (2000), and Leitner (2005)) ignore the tiered structure and do not analyse it in any rigorous way : "the notion that banks build yet another layer of intermediation between themselves goes largely unnoticed in the banking literature". Craig and von Peter (2010) find that the tiered character of this market is highly persistent. This could coincide with an outcome of competitive co-evolution in that to retain status quo in market shares, the core banks are hugely geared to the arms race involved there (see, also Galbiati and Giansante (2010)). Craig and von Peter (2010) go on to note that *"the persistence of this tiered structure poses a challenge to interbank theories that build on Diamond and Dybvig (1983). If unexpected liquidity shocks were the basis for interbank activity, should the observed linkages not be as random as the shocks? Should the observed network not change unpredictably every period? If this were the case, it would make little sense for central banks and regulatory authorities to run interbank simulations gauging future contagion risks. The stability of the observed interbank structure suggests otherwise."*

From our experience of mapping the financial networks based on actual bilateral data of FIs for the Indian financial system,[23] there appears to be a distinct variation in the core-periphery hierarchical structure noted by Craig and von Peter (2010) in the different types of financial activities. In their derivatives or contingent claims exposures and obligations, FIs show a far more marked concentration in the core both in terms of financial flows and connectivity, with a few banks in the core and a large number of them in the periphery. In non-contingent claims based borrowing and lending the interbank market shows more diffusion in the core with a larger number of banks in the core. The least hierarchical is the RTGS payment and settlements systems where there is a distinct lack of identifiable periphery banks. That the credit based interbank markets have different network properties to RTGS payment and settlement systems has also been noted by Kyriakopou-

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[23] See, Reserve Bank of India Financial Stability Report, December 2011.

los et. al. (2010).[24] Their findings on the network topology of the Austrian payment and settlement systems have been found to correspond to the study of the Fedwire payment and settlement system by Soramaki et al. (2006). Bech and Enghin (2008) did a detailed study of the network topology of Fed Funds market and found that the clustering of the system was limited and that small banks lend more to big banks than to their own sized banks showing disassortative linkages. They found that this disassortativity was reduced when links were weighted by value of flows. Hence, we emphasize the need for empirical calibrations that reflect actual market concentration in the financial activity or the use of full bilateral data on financial obligations between counterparties.

Finally, the presence of highly connected and contagion causing players typical of a clustered complex system network perspective is to be contrasted with what some economists regard to be an equilibrium network. Recently, Babus (2009) states that in "an equilibrium network the degree of systemic risk, defined as the probability that a contagion occurs conditional on one bank failing, is significantly reduced". Indeed, the premise of *TITF* is that the failure of a highly connected bank will increase the failure of another similarly bank, which we find to be the empirical characteristic of the network topology of the CDS market involving US banks, indicates that the drivers of network formation in the real world are different from those assumed in economic equilibrium models.

Our analysis of the stability of highly clustered financial networks has been influenced by the work of Robert May and studies on the spread of epidemics in non-homogenous networks with hierarchies (see, Kao (2010, p.62). May (1972, 1974) seminaly extended the Wigner condition of eigenvalues for complete random matrices to sparse random networks. He was the first to state that the stability of a dynamical network based system will depend on the size of the maximum eigenvalue of the weighted adjacency matrix of the network. Assuming the matrix entries are zero mean random variables, May (1974) derives the maximum eigenvalue of the network, which we denote as λ_{max}, in terms of three network parameters p, the probability of connectivity, N the number of nodes and σ which is the standard deviation of node

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[24] Note, as shown in Kyriakopoulos et al. (2009) the network mapping of electronic real time payment and settlement systems is highly sensitive to the time scale over which flows are estimated. This problem is not something that has been resolved yet.

strength. The May (1974) result states that network instability follows when $\sqrt{Np}\sigma > 1$. There is a trade off between heterogeneity in node strength, σ and connectivity, p, in order for the network to remain stable. In a non-zero mean random matrix, highly connected networks can remain stable only if they are homogenous in node strength, viz. σ should be very small. In networks with high variance to mean ratio in degrees and with tiered hierarchies of highly connected nodes where there is higher probability that a node is connected to a highly connected one, the direction of the epidemic which starts in a central hub follows a distinct hierarchical pattern with the highly connected nodes being infected first and the epidemic then cascading toward groups of nodes with smaller degrees, Kao (2010). Further, Kao (2010) notes that the epidemic dies out at great speed once the super-spreaders are eliminated. In contrast, in uncorrelated random graphs, the epidemic lasts longer and also reaches more nodes. For epidemic control, clustered networks enable targeting of specific individuals as opposed to inoculating the whole population in a random graph. Sinha (2005) and Sinha and Sinha (2006), also find that while both the small world and the Erdös-Renyi random graph show instability according to the condition given by May (1974), the lack of structure in a random graph results in a worse capacity of the system to cope with the contagion.

In terms of propagation of failure, therefore and as it will be shown, it is not true that financial systems where no node is too interconnected or involved in a cluster (as in an Erdös-Renyi random network) are necessarily easier to manage in terms of structural coherence and stability. Hence, we will report on the stability analysis of the empirically calibrated US CDS network and also of an equivalent random graph of the same size and functionality in terms of the CDS fair value flows. The instability propagation in the highly clustered empirically based CDS network and the equivalent random graph is radically different and the less interconnected system is in some respects more difficult to manage. This suggests the need for caution in espousing an ideal network topology for financial networks. This also underscores the importance of calibrations for networks in contagion analysis to be based on actual financial flows for the market or some close empirical proxies for network connectivity.

4. Contagion and Stability Analysis

The study of the topology of network in order to characterize its dynamical and stability properties has been actively studied especially in the context of ecology of species and in epidemiology. In financial network model the analysis of contagion from specific node failure has used the classic Furfine (2003) methodology.

4.1. Furfine (2003) Methodology : Failure of A Single Trigger bank at Initial Period

We follow the round by round or sequential algorithm for simulating contagion that is now well known from Furfine (2003). Starting with a trigger bank i that fails at time 0, we denote the set of banks that fail at each round or iteration by D^q, $q = 1, 2, \dots$. Note, the superscript q shows the q^{th} iteration. The cascade of defaults occur in the following way:

i Assuming tear ups but no novation of CDS contracts and zero recovery rate on the trigger bank i's liabilities, bank j fails if its direct bilateral net loss of CDS cover vis-á-vis the trigger bank i taken as a ratio of its capital (reported in the fifth column of Tables A.5, A.6 in the Appendix A) is greater than a threshold ρ. That is,

$$\frac{(x_{ij} - x_{ji})^+}{C_j} > \rho.$$

This threshold ρ signifies a percentage of bank capital which can be regarded as a sustainable loss. This is assumed to be the same for all banks.

ii A second order effect of contagion follows if there is some bank z, $z \notin D^1$, ie. those that did not fail in round 1, suffers losses due to counterparty failure such that the losses are greater than or equal to a proportion ρ of its capital:

$$\frac{\left[(x_{iz} - x_{zi})^+ + \sum_{j \in D^1} (x_{jz} - x_{zj})\right]}{C_z} > \rho.$$

The summation term aggregates the net loss of CDS cover to z from all banks $j, j \neq i$, which demised in the first iteration.

iii This then iterates to the q^{th} round of defaults if there is some bank v, $v \notin D^1 \cup D2 \cdots \cup D^{q-1}$, ie. has not failed till $q-1$, such that

$$\frac{\left[(x_{iv}-x_{vi})^{+}+\sum_{j\in\bigcup_{s}^{q-1}D^{s}}(x_{jv}-x_{vj})\right]}{C_v} > \rho.$$

iv The contagion is assumed to have ended at the round $q^{\#}$when there are no more banks left or none of those that have survived fail at $q^{\#}$.

4.2. Network Stability Analysis

Using the matrix $\boldsymbol{\Theta}$ in (2) whose entries give bilateral net liabilities of bank i to j as a ratio of bank j's capital, in matrix notation the equations for the dynamics of the cascade of failure given the failure of the trigger bank can be given as follows. Consider the column vector $\mathbf{U}_0$ with elements $(u_{1t}, u_{2t}, \ldots, u_{nt}) = (1, 0, \ldots, 0)$ to indicate the trigger bank that fails at initial date, $t = 0$, is bank 1 and the non failed banks assume 0's . The dynamics of bank failures is given by:

$$\mathbf{U}_{t+1} = \boldsymbol{\Theta}'\mathbf{U}_t - \rho\mathbf{I}. \tag{9}$$

Here, $\boldsymbol{\Theta}'$ is the transpose of the matrix in (2) and I is the identity matrix. Then, $\mathbf{U}_{t+1}$ gives the incremental failure of banks at $t+1$ (viz. banks that have not failed previously) where a binary function $F(z)$ sets each row u_{it+1} in the vector $\mathbf{U}_{t+1}$ to equal 1 to denote the failure of bank i and zero otherwise:

$$u_{it+1} = F\left(\sum_{j\in\mathbf{U}_t}\frac{x_{ji}-x_{ij}}{C_{it}} - \rho\right), F(z) = 1 \text{ if } z > 0 \text{ and } F(z) = 0 \text{ if } z \leq 0, \tag{10}$$

with $z = \sum_{j\in\mathbf{U}_t}\frac{x_{ji}-x_{ij}}{C_{it}} - \rho$.

Recall the elements of say the 2^{nd} row of Θ' takes the form:

$$\boldsymbol{\Theta}'_{2t} = \left(\frac{x_{12}-x_{21}}{C_{2t}}, 0, \frac{x_{32}-x_{23}}{C_{2t}}, \ldots, \frac{x_{N2}-x_{2N}}{C_{2t}}\right) \tag{11}$$

Here, each bank's capital evolves as:

$$C_{it+1} = C_{it} - \sum_{j\in\mathbf{U}_t}(x_{ji}-x_{ij}). \tag{12}$$

That is, bank i's capital at $t+1$ is reduced by the amount of the net obligations from its j counterparties that failed at t. At iteration $t+q$, the contagion is said to have halted if $\mathbf{U}_{t+q} = 0$. The number of failed banks at each t is given by:

$$N_t^{\#} = \sum_i u_{it}. \tag{13}$$

It is desirable that $\mathbf{U}_{t+q} = 0$ for some q before $N_{t+q}^{\#} = N$, the whole population is wiped out. The system stability of (9) is based on the largest positive real part of the eigenvalue of Θ' to be such that

$$\lambda_{max} < 1. \tag{14}$$

To derive this result, note from (9), $\mathbf{U}_{t+q}$ takes the form:

$$\mathbf{U}_{t+q} = \mathbf{\Theta}'^q \mathbf{U}_0 = \left(\sum_j \lambda_i^q \mathbf{v}_i' \mathbf{v}_i \right) \mathbf{U}_0. \tag{15}$$

Here ordering the N eigenvalues λ_i with $\lambda_1 = \lambda_{max} < \lambda_2 < \cdots < \lambda_N$ and denoting their respective eigenvectors as $\mathbf{v}_i$, if the contagion in (15) is to die off, for all i, λ_i^q should tend to zero and hence $\lambda_{max} < 1$.

4.3. Super-spreader Tax

Financial systems determined by matrix $\mathbf{\Theta}'$ that are prone to instability and contagion will have $\lambda_{max} > 1$. There are 4 ways in which stability of the financial network can be achieved: (i) Constrain the bilateral exposure of financial intermediaries; (ii) Increase the threshold ρ in (9); (iii) Change the topology of the network (iv) Levy a capital surcharge commensurate to the eigenvector centrality of a FI in (9). The first two measures do not price in the negative externality and systemic risk associated with failure of highly weighted network central nodes. Network topologies emerge endogenously and are hard to manipulate exogenously. The aim of the super-spreader tax is to have financial intermediaries with high eigenvector centrality parameters to internalize the costs that they inflict on others by their failure and to mitigate their impact on the system by reducing their contribution to network instability as given by λ_{max}. For this we use the well known eigenvector equation for $\mathbf{\Theta}'$:

$$\lambda_{max} = \mathbf{v}'\mathbf{\Theta}'\mathbf{v} = \sum_i \sum_j \theta_{ji} \text{ with } \sum_i v_i^2 = 1. \tag{16}$$

Critical to the von-Mises power iteration algorithm[25] for the calculation of λ_{max} and the corresponding eigenvector centrality v_i for node i is the row sum S_i of the i^{th} row in $\mathbf{\Theta}'$,

$$S_i = \sum_j \theta_{ji} = \frac{1}{C_i} \sum_j (x_{ji} - x_{ij})^+. \tag{17}$$

We create a new row sum $S_i^{\#}$, for each node so that a super-spreader tax denoted as $\tau(v_i)$ applies on the capital of the i^{th} node in proportion to its eigenvector centrality v_i :

$$S_i^{\#} = \sum_j \theta_{ji}^{\#} = \frac{1}{(1 + \tau(v_i))\, C_i} \sum_j (x_{ji} - x_{ij})^+. \tag{18}$$

Thus,

$$S_i^{\#} < S_i \text{ for } \tau(v_i) > 0. \tag{19}$$

We set the super-spreader tax :

$$\tau(v_i) = \alpha v_i^2,\ 0 < \alpha \leq 1 \text{ or } \alpha > 0. \tag{20}$$

The new matrix associated with $S_i^{\#}(\alpha)$, for all i, will be denoted as $\mathbf{\Theta}^{\#}(\alpha)$. Note, the super-spreader tax is set proportionate to v_i^2 rather than to v_i. This is because v_i^2 is a naturally normalized variable with $\sum_i v_i^2 = 1$. Further, with the super-spreader tax being a function of v_i^2 rather than v_i, this will penalize nodes with higher eigenvector centrality more than others. The alpha parameter when set at 0 obtains the λ_{max} associated with the untaxed initial matrix $\mathbf{\Theta}'$. When $\alpha = 1$, each node is exactly penalized by v_i^2, which yields the λ_{max} for $\mathbf{\Theta}'^{\#}(\alpha = 1)$. Considering, $0 < \alpha \leq 1$, there is a monotonic reduction in the λ_{max} associated with the matrices $\mathbf{\Theta}^{\#}(\alpha)$ corresponding to the monotonic reduction in row sums $S_i^{\#}(\alpha = 1) < \cdots < S_i^{\#}(\alpha = 0.75) < \cdots < S_i^{\#}(\alpha = 0.5) < \cdots < S_i(\alpha = 0)$. Remarkably, as will be seen, at $\alpha = 1$, λ_{max} for $\mathbf{\Theta}^{\#}(\alpha = 1)$ can be brought down to below 1 and

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[25] A detailed description of the algorithm is given in Ralston (1965).

the empirically calibrated CDS financial network for the US banks even in the absence of any pre-existing capital threshold can be stabilized. Clearly, the size of α, in particular if $\alpha > 1$ is needed to stabilize the system, the sustainability of such a market for risk sharing is in question.

The nature of the systemic risk stabilization super-spreader fund is that it operates like an escrow fund. The super-spreader taxes that are collected aim to cover the losses that the most connected nodes will inflict on their direct 'big' neighbours in the first tier. The empirical section will demonstrate the extent to which a super-spreader tax has to be levied in order to stabilize the system. It is designed to work in a clustered hierarchical network where contagion takes a specific pathway amongst the central tier if a highly connected node fails.

5. Empirical Results

5.1. Empirical (Small World) Network Algorithm

We study the US banks involved in the CDS market as recorded in the FDIC Call Reports for 2007 and 2008 Q4. In order to exclusively focus on the systemic risk from potential counterparty risk leading to loss of cover from CDS, FDIC data is obtained for CDS gross notional (buy and sell), Gross positive fair value (GNFV), Gross negative fair value (GNFV) and Tier 1 capital. Tables A.5, A.6 in the Appendix A reports the key data for 2007 and 2008 Q4.

As discussed, we use an algorithm that assigns network links on the basis of market shares (see, Tables A.5, A.6 in Appendix A) in order to reflect the very high concentration of network connections among the top 6 banks in terms of bilateral interrelationships. We first construct the X matrix given in (1). Our algorithm assigns in degrees and out degrees for a bank in terms of its respective market shares for gross notional values for CDS purchases and sales. Thus, in 2007 Q4 J.P. Morgan with a 50% share on both sides of the market will approximately have 15 in and out degrees. The choice of these 15 banks J.P. Morgan has out degrees to is assortative, i.e. 15 banks are chosen from the largest to the smallest in terms of their CDS activity.

- S_i^G : $Bank_i$ market share in terms of the gross notional on the sell side of CDS
- S_i^B : $Bank_i$ market share in terms of the gross notional on the buy side of CDS

- G_i : Gross Negative Fair Value for which $Bank_i$ is a guarantor vis-á-vis its counterparties

- B_i : Gross Positive Fair Value for which $Bank_i$ is beneficiary vis-á-vis its counterparties

The algorithm then allocates to each row bank i's counterparties j, a value of i's GNFV equal to $S^B_j G_i$ and if $\sum_j S^B_j G_i < G_i$, then bank i allocates the remaining to the external non-US bank entity which is the $N + 1$ agent. The column sums of matrix X in (1) are made to satisfy the $GPFV_j$ or B_j for each bank, the following allocation rule is used such that if $S^B_j \sum_i G_i < B_j$, the remaining is bought from the external entity.

In order to determine each bank's share of GNFV to the US non-bank sector which includes Monolines and hedge funds we use data from Table RCL-16a, "Derivatives and Off-Balance Sheet Items", from FDIC Call Reports which gives a sectoral break down. Finally, the share of a bank's GNFV for the entity called 'others' which denotes non-US counterparties is obtained as a balancing item to satisfy the condition given in (1) that $\sum_i G_i = \sum_j B_j$. The gross flow X matrix so constructed using the above algorithm is a sparse matrix with a very high concentration of activity. We then derive the bilaterally netted exposures between a pair of banks which can be read off accordingly as $(x_{ij} - x_{ji})$ with x_{ij} denoting GNFV for CDS protection from i to j and x_{ji} is GNFV protection cover from j to i. Hence, the size of bilateral net sell amount is given by $(x_{ij} - x_{ji}) > 0$. The resulting network for this is graphed below in Figure 3.

In Figure 3 red nodes denote net CDS sellers and blue nodes are net CDS buyers. The main difference between the US CDS networks for 2007 Q4 and 2008 Q4 is that the dominant role of the Monolines and hedge funds as net CDS sellers (largest red coloured node, LHS) has almost all been phased out by the end of 2008. By 2008 Q4 J.P. Morgan has increased its dominance as the sole member of the inner core and non-US banks (red triangle) become net protection providers. Hence, there are clear threats from the non-US sector, which we do not analyse. The other top 5 US banks remain in the central core of the network in somewhat weaker positions with the exception of Goldman Sachs which migrates more to the centre in 2008 Q4. Over 80% of the banks are in the periphery with almost no connectivity among themselves manifesting a very sparse adjacency matrix.

The tiered layout in Figure 3 is constructed in the following way. We take the range of connectivity of all banks as a ratio of each bank's total in and

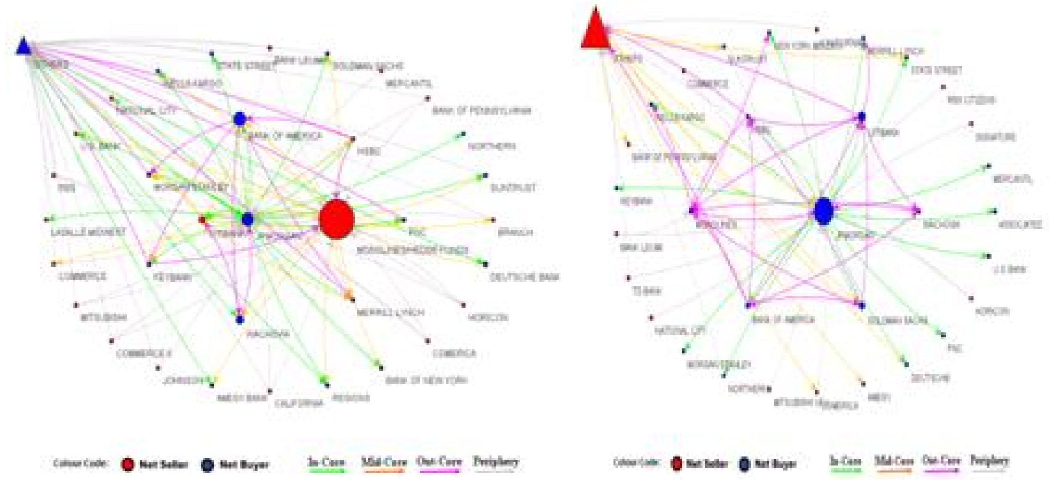


Figure 3: The Empirically Constructed CDS Network (Bilaterally Netted) for US Banks and Non-US Financial Intermediaries (Triangle): Empirical Small World Network in Tiered Layout (LHS 2007 Q4 and RHS 2008 Q4).

out degrees divided by that of the most connected bank. Banks that are a ranked in the top 10 percentile of this ratio constitute the inner core. This is followed by a mid core between 90 and 70 percentile and a 3rd tier between 40 and 70 percentile. Those with connectivity ratio less than 40 percentile are categorized as the periphery.

The links are weighted and thicker the links, the larger the size of their obligations. The links are colour coded. The triangle entity representing non-US banks constitutes the mid-core. So the yellow links show where the second tier (mid core) banks are offering protection. As can be seen, the banks with the pink arrows in the core almost always interact with one another. In 2008 Q4 the largest size rich club with a coefficient of 1 (defined in equation (5)) has only 4 members. The banks in the periphery are mostly sold protection by J.P. Morgan.

Table 1 gives the network statistics for the empirically constructed CDS networks and also for the equivalent random graph representing the 2008 CDS data given in Figure 4. The random graph is constructed with the same connectivity of about 6% as the market share based empirically constructed network for 2008 Q4 (see, Appendix B for the algorithm used in the construction of the random graph.) The main difference in the network statistics for the 2007 Q4 and 2008 Q4 CDS networks is the jump in the

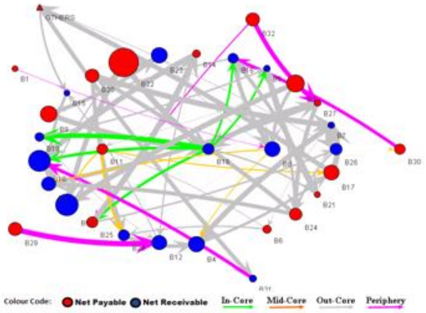
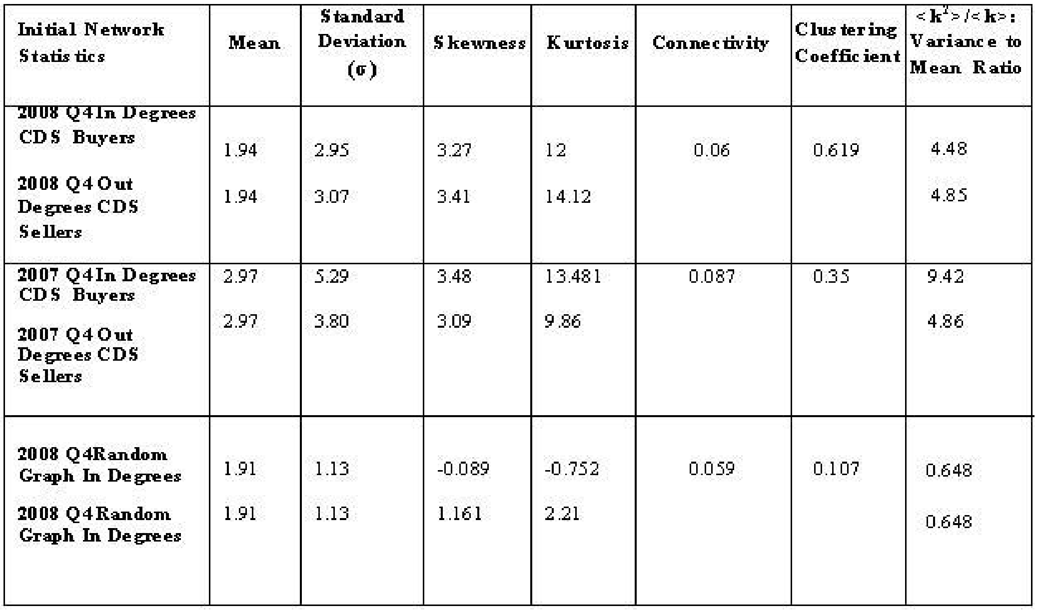


Figure 4: Erdös-Renyi Random Graph (Equivalent to 2008 Q4 CDS Network in Figure 3 RHS) for US Banks and Non US Sector (Triangle): Absence of an identifiable core-periphery structure

clustering coefficient in 2008 Q4 to 62% from 35% while connectivity has fallen from about 8% to 6%. The random graph has a much lower cluster coefficient of 10% compared to that of about 62% for the empirical CDS network based on the 2008 Q4 data. Also, the random graph has substantially low variance to mean ratio than the empirically calibrated CDS networks. The highly asymmetric nature of the empirical CDS network is manifested in the large kurtosis or fat tails in degree distribution which is characterized by a few (two banks in this case) which have a relatively large number of in degrees (up to 14) while many have only a few (as little as 1).

5.2. Eigenvector Centrality and Furfine Stress Test Results

Here we will investigate the idea about the role of super-spreaders of contagion in terms of their network connectivity, dominance as CDS protection sellers and their weighted eigenvector centrality. As already noted, in the post Lehman era of 2008 Q4, the dominance of J.P. Morgan is the key aspect of the US sector of the CDS market. In terms of connectivity, J.P. Morgan stands out by a large margin with 55% share of total out degrees. Citi has 12.5% of outdegreees while Goldman Sachs and HSBC come in at third place with a modest 9.3% share. In terms of eigenvector centrality which correlates best with contagion losses the trigger bank inflicts on others, again J.P. Morgan with eigen vector centrality of 0.63 seems to be the only bank with substantial systemic risk consequences. Goldman Sachs comes second with eigenvector centrality of 0.54 and HSBC third with 0.38. This is borne out in the Furfine stress tests results given in Table 2 and Figure 5. Over all, J.P. Morgan as trigger bank results in the failure of Morgan Stanley, Citigroup,



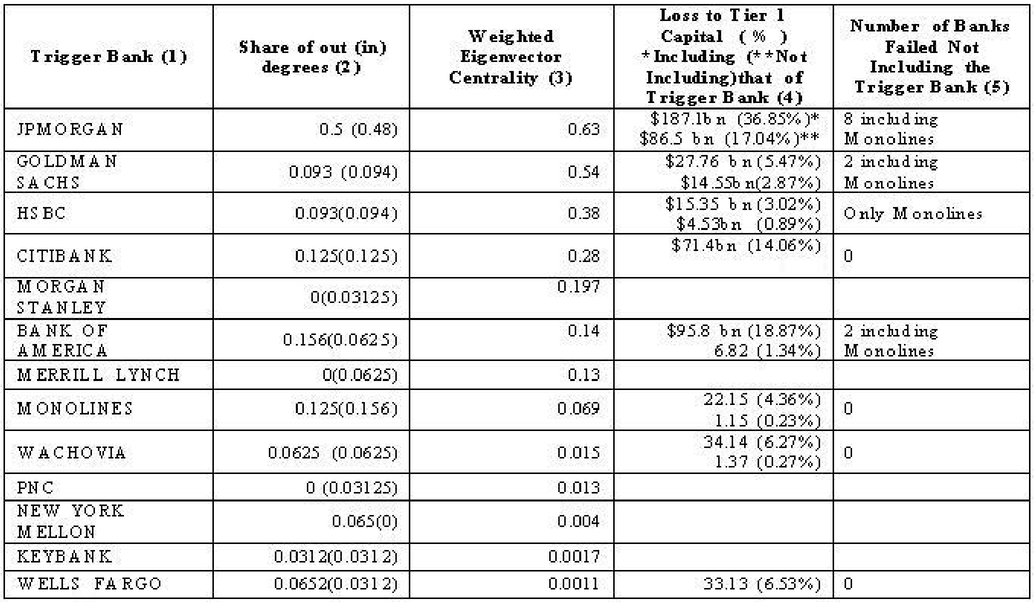
Initial Network Statistics	Mean	Standard Deviation (σ)	Skewness	Kurtosis	Connectivity	Clustering Coefficient	$<k^2>/<k>$: Variance to Mean Ratio
2008 Q4 In Degrees CDS Buyers	1.94	2.95	3.27	12	0.06	0.619	4.48
2008 Q4 Out Degrees CDS Sellers	1.94	3.07	3.41	14.12			4.85
2007 Q4 In Degrees CDS Buyers	2.97	5.29	3.48	13.481	0.087	0.35	9.42
2007 Q4 Out Degrees CDS Sellers	2.97	3.80	3.09	9.86			4.86
2008 Q4 Random Graph In Degrees	1.91	1.13	-0.089	-0.752	0.059	0.107	0.648
2008 Q4 Random Graph In Degrees	1.91	1.13	1.161	2.21			0.648

Table 1: Network Statistics for Degree Distribution for CDS Network: Small World Network Properties Compared with Random Graph with Same Connectivity

Bank of America, Goldman Sachs, HSBC and Merrill Lynch in the first tier of the network.

5.3. Contagion: Clustered Small World vs Random CDS Network

We will compare the CDS network stability of a random graph of the same size, connectivity and gross flow functionalities with that of the more clustered empirically based CDS network. Some very interesting issues are highlighted here as discussed in Section 4. Recall the marked difference in structure is the clustering coefficient of the two networks and high variance to mean ratios (see, Table 1). The high clustering of the small world network in regard of what we understand to be the most likely structure for the CDS network in order to reflect the high concentration of exposures between 5 or so counterparties, displays a distinct pattern of propagation of financial contagion from the demise of the dominant bank, J.P. Morgan. As shown in Figure 5 (LHS) in the clustered network, there are only direct failures in a closed sector rather than higher order failures spreading to the whole system. It is, ofcourse, cold comfort that the first order shock wipes out the top 5 banks. Together they lead to the failure of the non-bank US CDS



Trigger Bank (1)	Share of out (in) degrees (2)	Weighted Eigenvector Centrality (3)	Loss to Tier 1 Capital (%) *Including (**Not Including)that of Trigger Bank (4)	Number of Banks Failed Not Including the Trigger Bank (5)
JPMORGAN	0.5 (0.48)	0.63	\$187.1bn (36.85%)* \$86.5 bn (17.04%)**	8 including Monolines
GOLDMAN SACHS	0.093 (0.094)	0.54	\$27.76 bn(5.47%) \$14.55bn(2.87%)	2 including Monolines
HSBC	0.093(0.094)	0.38	\$15.35 bn(3.02%) \$4.53bn (0.89%)	Only Monolines
CITIBANK	0.125(0.125)	0.28	\$71.4bn (14.06%)	0
MORGAN STANLEY	0(0.03125)	0.197		
BANK OF AMERICA	0.156(0.0625)	0.14	\$95.8 bn(18.87%) 6.82 (1.34%)	2 including Monolines
MERRILL LYNCH	0(0.0625)	0.13		
MONOLINES	0.125(0.156)	0.069	22.15 (4.36%) 1.15 (0.23%)	0
WACHOVIA	0.0625 (0.0625)	0.015	34.14 (6.27%) 1.37 (0.27%)	0
PNC	0 (0.03125)	0.013		
NEW YORK MELLON	0.065(0)	0.004		
KEYBANK	0.0312(0.0312)	0.0017		
WELLS FARGO	0.0652(0.0312)	0.0011	33.13 (6.53%)	0

Table 2: 2008 Q4 Eigenvector Centrality and Furfine Stress Tests (for selected banks) with capital threshold.

users. In contrast, in the random graph, while no node is either too big or too interconnected, the substantial part of the system unravels (up to 17 banks fail) in a series of multiple knock on effects. Note the concentric circles denote the sequence of cascade or iteration q described in section 4.1. The black nodes are the failed banks and the green ones are those that are hit but do not fail.

5.4. Quantification and Evaluation of the Super-spreader Tax (2008 Q4)

At a maximum eigenvalue of 1.157, the system is deemed unstable and the losses to the system as a whole from the failure of the eigenvector dominant bank, J.P. Morgan, will be nothing short of catastrophic with the failure of 5 top banks (see Figure 5 LHS). Socialized losses have to be internalized by the banks themselves. In this section, we will evaluate the super-spreader tax based on a theoretical derivation in Section 4.3 and equation (20). A surcharge on bank capital commensurate to the eigenvector centrality of a bank using the formula in equation (20) $\tau(v_i) = \alpha v_i^2$ is applied to the rows of $\mathbf{\Theta}'$ for different values of $0 < \alpha \leq 1$. Note the eigenvector centrality for the top 13 banks is given in Table 2. This results in

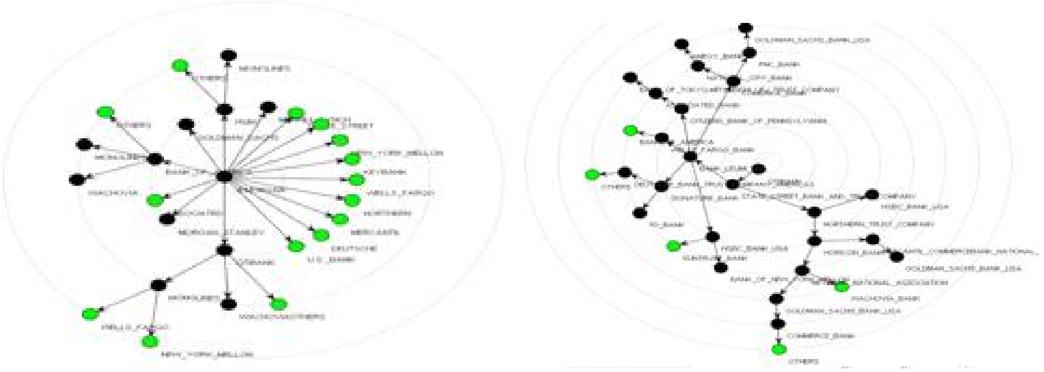


Figure 5: Instability propagation in Clustered CDS Network(2008 Q4 LHS) and in Equivalent Random Network (RHS) NB: Concentric circles mark the iterations q given in section 4.1; failed banks are black nodes and green nodes are those that are 'hit' but do not fail.

a monotonic reduction in the maximum eigenvalue $\lambda^{\#}_{max}$ associated with the matrices $\mathbf{\Theta}'^{\#}(\alpha)$ corresponding to the monotonic reduction in row sums $S^{\#}_i(\alpha = 1) < \cdots < S^{\#}_i(\alpha = 0.75) < \cdots < S^{\#}_i(\alpha = 0.5) < \cdots < S_i(\alpha = 0)$. This is given in Figure 6 and Table 3.

Remarkably, the $\lambda^{\#}_{max}$ of the system can be brought to a touch under 1 with $\alpha = 1$. What is interesting to note is that while the stability of the system will improve as evidenced in the reduction of the maximum eigenvalue of the matrix as banks hold more capital commensurate with their eigenvector centrality given by the original untaxed matrix $\mathbf{\Theta}'$, as Table 3 shows, this is by far the main manifestation of the impact of the super-spreader surcharge. There is some decrease in the skewness and kurtosis of the distribution of eigenvector centrality of the nodes in the matrix $\mathbf{\Theta}'(\alpha = 0.9, \alpha = 1$, see Table 3).

Figure 7 gives the rate of super-spreader surcharge and that needs to be levied on the banks in order that they internalize the systemic risk costs arising solely from their network centrality. The super-spreader tax rate is obtained by multiplying the square of eigenvector centrality of each node v_i^2 by the alpha parameter given in equation (20) which then brings the λ_{max} of the matrix $\mathbf{\Theta}'$ to below 1. Table 4 will focus on the case of when $\alpha = 1$ and how super-spreader escrow fund will stabilize the system. It is important to see if the super-spreader escrow fund can obtain sufficient funds which can cover the Tier 1 capital losses sustained (\$67 bn in the absence of any pre-existing threshold and \$55 bn if a 6% threshold exists) when the

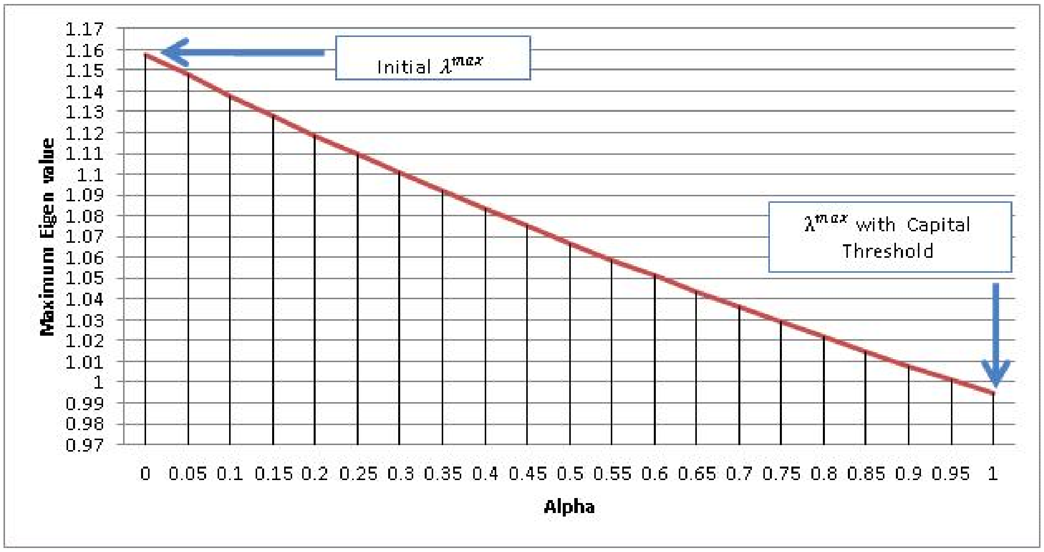


Figure 6: Maximum Eigenvalue ($\lambda^{\#}_{max}$) for different values of α (Equation 20): Note the initial $\lambda_{max} = 1.157$.

α	$\lambda^{\#}_{max}$	Mean eigenvector centrality for nodes	Standard Deviation eigenvector centrality for nodes	Skewness	Kurtosis	Min Eigenvector centrality for nodes	Max Eigenvector centrality for nodes
0	1.158	0.078	0.16	2.39	5.1	1.19E-07	0.63
0.05	1.148	0.078	0.16	2.38	5.07	1.18E-07	0.63
0.1	1.138	0.078	0.16	2.37	5.04	1.17E-07	0.63
0.3	1.10	0.078	0.16	2.35	4.92	1.12E-07	0.63
0.4	1.08	0.079	0.16	2.33	4.86	1.1E-07	0.63
0.5	1.067	0.079	0.16	2.32	4.81	1.08E-07	0.63
0.6	1.05	0.079	0.16	2.31	4.76	1.06E-07	0.63
0.7	1.036	0.079	0.16	2.29	4.72	1.04E-07	0.63
0.8	1.025	0.079	0.16	2.29	4.67	1.03E-07	0.63
0.9	1.008	0.080	0.16	2.28	4.63	1.01E-07	0.63
1	0.99	0.080	0.16	2.27	4.59	9.96E-08	0.63

Table 3: Maximum Eigenvalue $\lambda^{\#}_{max}$ for the Sequence of Matrices $\mathbf{\Theta}'^{\#}(\alpha)$ for $0 < \alpha \leq 1$.

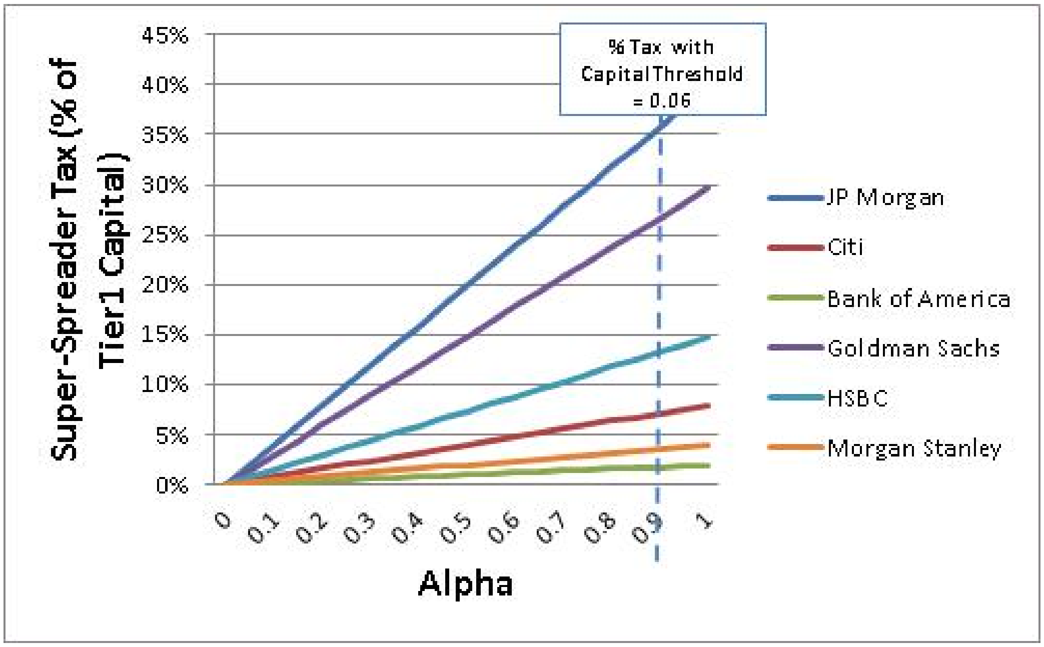
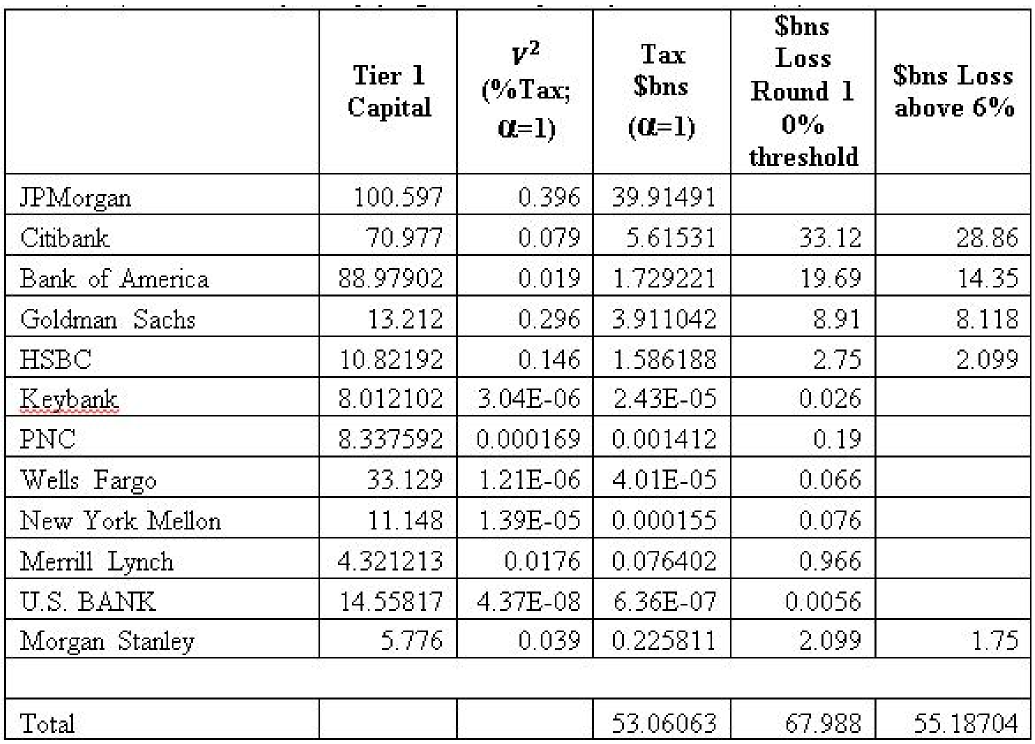


Figure 7: Super-spreader Tax Rates On Banks to Achieve Different Levels of Network Stability (Calibrated using different α given in equation (20))

most eigen-vector dominant bank J.P. Morgan fails. What is clear from the analyses is that only up to six banks need to be levied a non-zero tax on the basis of their network centrality parameter to fully price in the potential threat to the tax payer if they fail. As shown in Figure 7 and Table 4, J.P. Morgan's capital surcharge stands at 37%, 28% for Goldman Sachs, 13% for HSBC, 7% for Citigroup and under 3% for Morgan Stanley and about 2% for Bank of America. Table 4 gives the amounts that will accrue in the super-spreader fund and we verify that this will cover over 95% of the losses that will be incurred by the demise of the 5 top tier banks due to the failure of the dominant eigenvector central bank J.P. Morgan. Table 4 also indicates the Tier1 capital losses that need to be recovered as they are negative externalities in round 1 from the failure of J.P. Morgan both in the case of zero threshold ($\rho = 0$)and when the threshold $\rho = 0.06$.

6. Concluding Remarks

This paper investigated the systemic risk posed by the topological fragility of the CDS market due to the concentration in CDS exposures between few



	Tier 1 Capital	V^2 (%Tax; α=1)	Tax $bns (α=1)	$bns Loss Round 1 0% threshold	$bns Loss above 6%
JPMorgan	100.597	0.396	39.91491		
Citibank	70.977	0.079	5.61531	33.12	28.86
Bank of America	88.97902	0.019	1.729221	19.69	14.35
Goldman Sachs	13.212	0.296	3.911042	8.91	8.118
HSBC	10.82192	0.146	1.586188	2.75	2.099
Keybank	8.012102	3.04E-06	2.43E-05	0.026	
PNC	8.337592	0.000169	0.001412	0.19	
Wells Fargo	33.129	1.21E-06	4.01E-05	0.066	
New York Mellon	11.148	1.39E-05	0.000155	0.076	
Merrill Lynch	4.321213	0.0176	0.076402	0.966	
U.S. BANK	14.55817	4.37E-08	6.36E-07	0.0056	
Morgan Stanley	5.776	0.039	0.225811	2.099	1.75
Total			53.06063	67.988	55.18704

Table 4: Super-spreader Tax Fund (Total and selected banks) and Value of Round 1 Tier 1 Capital Losses (Super-spreader Tax ($bns) calculated by multiplying Tier1 capital by the tax rate (%)).

highly connected US banks. To date, till the work of Craig and von Peter (2010), financial network modellers have failed to sufficiently focus on the core-periphery structure of financial intermediaries. A large number of financial network models have either assumed a Erdös-Renyi random network structure (see, Nier et al. (2007)) or complete graphs constructed by entropy methods. The entropy based models are known not to produce financial contagion with the failure of any trigger bank (see, Upper and Worms (2004)). The core-periphery tiered network is particularly relevant for derivatives markets. The framework we use to build an empirically based network for the CDS obligations between US banks and non-banks reveals the high clustering phenomena of small world networks along with a sparse adjacency matrix. We used the market share of CDS activity by banks to determine the network structures as discussed above.

We have characterized *TITF* phenomena of the CDS market with the tiered structure given in Figure 3. The 2008 Q4 CDS network is seen to have substantially more clustering than in 2007 Q4 and gives evidence of the greater concentration of CDS exposures among even fewer US banks than in 2007. The clustered network as seen in Figure 4 showed the radically different way in which contagion propagates in contrast with an Erdös-Renyi network. This is well understood in network models of epidemics, but not so much in financial models. Clustered small world network structure has some capacity for containment of contagion and in complex system terms these highly interconnected multi-hub based systems can have some stabilizing effects compared to the unstructured random graphs. However, it is clear that the increased capacity to bear the first order shocks by the hub entities could only be achieved by installing 'super-spreader reserves', overturning the current practice of leniency in this direction.

The financial network implied by the bilateral exposures given in a matrix such as $\mathbf{\Theta}'$ in section 4 is examined for its stability in terms of its maximum eigenvalue. We found the empirically calibrated CDS network for the bilaterally netted exposures for the US FDIC banks for 2008 Q4 has maximum eigenvalue of 1.15. The network shows that J.P. Morgan is the most dominant bank in regard to eigenvector centrality, followed by a long margin by Goldman Sachs and HSBC. In order for banks to internalize the systemic risk from the failure of banks that have high network centrality, we recommend that banks be taxed by a progressive tax rate based on the square of their eigenvector centrality and to escrow the tax surcharge. This is the first operationalization of this concept with the application of the super-spreader tax demonstrated to reduce the maximum eigenvalue of the matrix of netted liabilities of financial intermediaries. We ‘back tested’ the capacity of this fund to cover the maximum losses from the most network central bank. The stability analysis is one that can be used to evaluate the adequacy of the amounts of collateral or capital to absorb losses from a potential failure of counterparties even in a Central Clearing Platform without tax payer bailouts. Further experimentation with a multi-agent financial network model is needed to answer questions such as: how well will the super-spreader tax fund perform, one which is based only on unweighted eigenvector centrality of the financial intermediaries which requires much less information ? How will banks change their behaviour when faced by the full cost of being *TITF*? Can the super-spreader tax be applied and altered like a traffic congestion pricing scheme as behaviour of agents adopts? [26]

It is our view that the size of derivatives markets and CDS markets, in particular, far exceed their capacity to internalize the potential losses that follow from the failure of highly connected financial intermediaries. The large negative externalities that arise from a lack of robustness of the CDS financial network from the demise of a big CDS seller further undermines the justification in Basel II and III that banks be permitted to reduce capital on assets that have CDS guarantees. We recommend that the Basel II provision for capital reduction on bank assets that have CDS cover should be discontinued. Banks should be left free to seek unfunded CDS cover for bank assets without the incentive of capital reduction and leverage. Indeed, this may enhance price discovery role of the CDS market relating to the probability of default of reference assets or entities.

Appendix A. FDIC Data

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[26] See Markose et al. (2007) for an agent based model to price and monitor congestion in a real world application.

Name	Gross Notional CDS Buy		Gross Notional CDS Sell		GPFV		GNFV		Tier 1 Core Capital
JPMORGAN	4016.58	51%	3860.57	50%	130.19	44%	126.08	44%	78.45
BANK OF AMERICA	1483.96	19%	1522.46	20%	55.88	19%	51.25	18%	75.40
CITIBANK	1610.32	20%	1505.62	19%	76.59	26%	78.08	27%	81.95
HSBC	586.65	7%	638.07	8%	19.71	7%	19.79	7%	9.70
WACHOVIA	179.63	2%	188.35	2%	14.53	5%	12.65	4%	40.47
KEYBANK	4.35	0%	3.33	0%	0.07	0%	0.06	0%	7.14
PNC	3.96	0%	2.10	0%	0.10	0%	0.06	0%	7.85
WELLS FARGO	1.88	0%	0.87	0%	0.08	0%	0.03	0%	29.55
NATIONAL CITY	1.38	0%	0.85	0%	0.01	0%	0.01	0%	8.36
SUNTRUST	0.78	0%	0.31	0%	0.02	0%	0.02	0%	12.34
MITSUBISHI UFJ	0.01	0%	0.15	0%	0.00	0%	0.02	0%	0.78
REGIONS	0.07	0%	0.13	0%	0.01	0%	0.00	0%	9.80
COMMERCE	0.00	0%	0.07	0%	0.00	0%	0.00	0%	2.49
BRANCH BANKING AND TRUST	0.01	0%	0.07	0%	0.00	0%	0.00	0%	8.47
COMMERCE	0.00	0%	0.03	0%	0.00	0%	0.00	0%	1.15
RBS CITIZENS	0.00	0%	0.02	0%	0.00	0%	0.00	0%	7.93
BANK LEUMI	0.00	0%	0.01	0%	0.00	0%	0.00	0%	0.42
JOHNSON	0.00	0%	0.01	0%	0.00	0%	0.00	0%	0.31
COMERICA	0.01	0%	0.01	0%	0.00	0%	0.01	0%	5.73
HORICON	0.00	0%	0.01	0%	0.00	0%	0.00	0%	0.04
CITIZENS BANK OF PENNSYLVANIA	0.00	0%	0.00	0%	0.00	0%	0.00	0%	2.26
BANK OF NEW YORK	2.09	0%	0.00	0%	0.04	0%	0.00	0%	6.46
CALIFORNIA BANK & TRUST	0.00	0%	0.00	0%	0.00	0%	0.00	0%	0.69
AMEGY BANK	0.00	0%	0.00	0%	0.00	0%	0.00	0%	0.74
MORGAN STANLEY	15.32	0%	0.00	0%	0.27	0%	0.03	0%	3.15
DEUTSCHE BANK	0.10	0%	0.00	0%	0.38	0%	0.02	0%	8.49
MERCANTIL COMMERCE BANK	0.01	0%	0.00	0%	0.00	0%	0.00	0%	0.44
STATE STREET BANK	0.24	0%	0.00	0%	0.00	0%	0.00	0%	6.91
U.S. BANK	0.06	0%	0.00	0%	0.00	0%	0.00	0%	13.21
GOLDMAN SACHS	0.56	0%	0.00	0%	0.01	0%	0.00	0%	1.21
MERRILL LYNCH	8.73	0%	0.00	0%	0.23	0%	0.01	0%	6.51
NORTHERN TRUST	0.28	0%	0.00	0%	0.00	0%	0.00	0%	3.02
LASALLE BANK MIDWEST	2.10	0%	0.00	0%	0.00	0%	0.00	0%	2.05
AGGREGATE	**7919.07**		**7723.03**		**298.11**		**288.13**		**443.48**

Table A.5: FDIC Data (2007 Q4) for 33 US Banks With CDS Positions ($ bn)

Name	Gross Notional CDS Buy		Gross Notional CDS Sell		GPFV		GNFV		Tier 1 Core Capital
JP Morgan Chase	4166.76	53%	4199.10	54.3%	538.87	48%	455.56	46%	100.61
Citibank	1397.55	18%	1290.31	16.7%	211.65	19%	188.43	19%	70.98
Bank of America	1028.65	13%	1004.74	13.0%	132.04	12%	123.75	13%	88.50
Goldman Sachs	651.35	8%	614.40	7.9%	144.67	13%	131.75	13%	13.19
HSBC	457.09	6%	473.63	6.1%	64.83	6%	64.49	7%	10.81
Wachovia	150.75	2%	141.96	1.8%	24.08	2%	23.35	2%	32.71
Morgan Stanley	22.06	0%	0.00	0.0%	2.13	0%	0.03	0%	5.80
Merrill Lynch	8.90	0%	0.00	0.0%	1.19	0%	0.02	0%	4.09
Keybank	3.88	0%	3.31	0.0%	0.19	0%	0.17	0%	8.00
PNC	2.00	0%	1.05	0.0%	0.29	0%	0.09	0%	8.34
National City	1.29	0%	0.94	0.0%	0.00	0%	0.01	0%	12.05
The Bank of NY Mellon	1.18	0%	0.00	0.0%	0.08	0%	0.00	0%	11.15
Wells Fargo	1.04	0%	0.49	0.0%	0.15	0%	0.08	0%	33.07
SunTrust	0.59	0%	0.20	0.0%	0.26	0%	0.24	0%	12.56
The Northern Trust Company	0.24	0%	0.00	0.0%	0.04	0%	0.00	0%	4.39
State Street Bank and Trust Company	0.15	0%	0.00	0.0%	0.11	0%	0.00	0%	13.42
Deutsche Bank Trust Company Americas	0.10	0%	0.00	0.0%	0.00	0%	0.00	0%	7.87
Regions Bank	0.08	0%	0.41	0.0%	0.00	0%		0%	9.64
U.S. Bank	0.06	0%	0.00	0.0%	0.01	0%	0.00	0%	14.56
Commerce Bank	0.02	0%	0.03	0.0%	0.00	0%	0.00	0%	1.37
Mercantil Commercebank	0.01	0%	0.00	0.0%	0.00	0%	0.00	0%	0.54
Associated Bank	0.01	0%	0.12	0.0%	0.00	0%	0.00	0%	1.58
Comerica Bank	0.01	0%	0.05	0.0%	0.01	0%	0.03	0%	5.66
Signature Bank	0.00	0%	0.00	0.0%	0.00	0%	0.00	0%	0.76
RBS Citizen	0.00	0%	0.06	0.0%	0.00	0%	0.00	0%	8.47
Bank of Tokyo-UF	0.00	0%	0.05	0.0%	0.00	0%	0.04	0%	0.70
Aggregate	**7893.77**		**7730.85**		**1120.60**		**988.04**		**480.82**

Table A.6: FDIC Data (2008 Q4) for 27 US Banks With CDS Positions ($ bn)

Appendix B. Random Network Algorithm

The algorithm that creates a random network of CDS obligations proceeds on the following steps:

1. An adjacency matrix $\mathbf{A}(N \times N)$ is created where each element has value 1 with probability p (this probability is set to be equal to the connectivity of the empirical network we want to compare with), 0 otherwise.
2. A matrix $\mathbf{R}(N \times N)$ of random numbers is created where each element r_{ij} is randomly drawn from an uniform distribution in the range $[0, 1]$.
3. The matrix $\mathbf{B}(N \times N)$ of random values is generated as follows: $B = A{*}R$(element by element multiplication). The matrix $\mathbf{B}$ is now a sparse matrix with many zero elements.
4. The final flow matrix corresponding to X in equation (1) of CDS obligations $\mathbf{X}$ is defined as

$$\mathbf{X} = \mathbf{B}\frac{\Gamma}{\sum_i \sum_j b_{ij}}$$

. Here, Γ is the total CDS GNFV in the market as required by the empirically constructed matrix

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